GOAL PROGRAMMING APPROACH TO MULTI-OBJECTIVE OPTIMIZATION OF SURFACE GRINDING OPERATIONS

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

By
M. M. JOSHI

to the

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

APRIL, 1981

ME-1981-M-JOS-GOA

DEDICATED TO MY

RESPECTED PARENTS

28/11/81

CERTIFICATE

Certified that the work entitled "GOAL PROGRAMMING APPROACH TO MULTIOBJECTIVE OPTIMIZATION OF SURFACE GRINDING OPERATIONS" by Madan Mohan Joshi has been carried out under our supervision and has not been submitted elsewhere for a degree.

(Dr. G.K. Lal)	(Dr. J.L. Batra)
Professor	Professor
Deptt. of Mechanical Engg.	Department of Mech. Engg.
Indian Institute of Technology	Indian Institute of Technology
Kanpur	Kanpur

ACKNOWLEDGEMENT

I express my deep sense of gratitude and sincere thanks to Dr. J.L. Batra and Dr. G.K. Lal for their kind, able and dynamic guidance in the inception, execution and completion of this thesis.

I am very much indebted to Dr. Sada Gopan for his generous help and suggestions.

My thanks are due to Drs. S.J. Pande and G. Srihari for providing their experimental data for this work.

My sincere thanks are due to Mr. R.C. Gupta and Mr. A.K. Srivastava for many valuable discussions and suggestions.

I wish to express my deep sense of respect to Dr. J.L. Gaindhar for his constant affection and encouragement.

I thank all of my friends who have made my stay at I.I.T. Kanpur pleasant.

Finally, I take the opportunity to thank Mr. J.D. Varma for excellent typing.

Madan Mohan Joshi

TABLE OF CONTENTS

NOMENCLATURE

b	:	Width of Workpiece
b'	:	Width of workpiece
B	:	Width of Wheel
B_1	:	Traverse length of diamond dresser across wheel face in a single pass
c	:	Number of active grains per unit area
C_o	:	Operating cost per unit time
C_w	:	Wheel cost per unit volume
C_1	:	Truing and wheel segment replacement cost
C_g	:	Machining cost
C	:	Total cost per piece
d_d	:	Dressing depth of cut
d_g	:	Reduction in wheel diameter during grinding
d_t	:	Total depth of cut to be removed per piece
d_1	:	Total depth of material required to be removed
d_T	:	Depth of cut given during truing operation
D	:	Wheel diameter
F_n	:	Normal grinding force
F_t	:	Tangential grinding force
F_l	:	Longitudinal grinding force
G	:	Grinding ratio
m	:	Number of passes required for truing
m_1	:	Number of passes required for sparkout
m_2	:	Number of passes required for dressing
d	:	Depth of cut per pass

L_1	:	Length of stroke
L	:	Length of workpiece
L_o	:	Clearance length
t_L	:	Non-productive time
TC	:	Total cost per piece
T_L	:	Non-productive time
T	:	Wheel life
v	:	Work velocity (Table speed)
V	:	Wheel velocity
v_c	:	Cross-feed rate during truing and dressing operations

Other notations used are described as and when used.

SYNOPSIS

GOAL PROGRAMMING APPROACH TO MULTI-OBJECTIVE OPTIMIZATION OF SURFACE GRINDING OPERATIONS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY
by

Madan Mohan Joshi
to the
DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

April 1981

The determination of optimum cutting parameters in grinding involves consideration of multiple objectives which are conflicting in nature. For example, in the fine surface grinding operation, the optimum set of cutting parameters should be such that not only the total cost of grinding per piece is minimized but certain levels of grinding wheel life and metal removal rate are also achieved. In this thesis, the problem of determining the optimum grinding parameters under conflicting objective environment is formulated as a goal programming problem. The models for horizontal and vertical surface grinding operations, the typical representative of fine and stock removal grinding processes, are developed and solved using a partitioning algorithm developed by Arthur and Ravindran [22] for solving

linear goal programming problems. Since the attainment of desired wheel life is not a relevant goal in vertical surface grinding, another goal which corresponds to grinding ratio has been considered. The various goals are considered in a hierarchial fashion and are achieved as closely as possible considering the various constraints imposed by the machine tool and wheel-work combination. The extensive experimental data obtained by Pande [18] and Srihari [37] are analyzed to establish empirical relationships between wheel life, grinding ratio and grinding forces and the cutting conditions (work velocity and depth of cut). Logarithmic transformation has been used for linearizing the various goals and constraints, so that model becomes amenable for solution using the linear goal programming technique.

A typical example for each of the two grinding cases is considered for the illustration of the models. The results obtained using the partitioning algorithm are presented.

CHAPTER 1

INTRODUCTION AND LITERATURE SURVEY

1.1 INTRODUCTION

Machining is one of the most important manufacturing processes. Machining costs usually represent a very significant portion of finished component cost and hence a feasible procedure for manufacture of the desired component should be based on both technological and economic considerations. The variables affecting the economics of a machining operation are numerous and include tool material characterstics, machine-tool capabilities and cutting conditions.

In 1907, Taylor [1] recognized the problem of economic machining in the metal cutting field in his pioneering work "On the Art of Cutting Metals". Further studies on economics of metal machining were carried out by Gilbert et al. [2] . Since then it is generally considered that determination of economic cutting conditions is a conflict between maximizing the metal removal rate and minimizing the tool wear. By increasing the feed rate or speed, the metal removal rate and hence the production rate can be increased, but this results in excessive tool wear, more frequent tool changes and increased cost of production. The concept of Hi - E Range (High Efficiency Range) which gives a compromise between the minimum cost and maximum production, was also developed by Gilbert's group. The optimum machining

condition belongs to the set which balances these conflicts as well as stays within the restrictions such as machine - tool capabilities and surface quality of the component.

1.1.1 Optimization in Metal Cutting

Recently there have been a number of attempts to apply Mathematical Programming Techniques for optimizing the cutting conditions in machining processes with respect to a single objective. Most of these attempts have been, however, confined to turning and milling operations and operations such as surface grinding have been more or less ignored. The general form of the economic machining problem can be stated as follows:

$$\text{Minimize} \quad f\,(x)$$
$$\text{Subject to} \quad g\,(x) \leq 0$$

where x is a set of machining parameters, $g\,(x)$ is a set of restrictions on horse power, temperature, surface finish and other cutting conditions and $f\,(x)$ is the objective function chosen. The solution of this mathematical optimization problem, in the case of single-pass, turning operation, has been attempted using differential calculus [3], Lagrange multipliers [4, 5], geometric programming [6], linear programming [7] & nonlinear programming [8].

In certain production situations single objective optimization may not be appropriate since the management

may wish to consider several criteria for determining the optimal solution to the problem. Often the various objectives may be competing and it may be impossible to obtain the minima for all of them simultaneously. Hence, any solution must include some compromise amongst the minima of these objectives. This type of situation occurs in optimization of manufacturing processes. For example, in the turning operation such a situation occurs in selecting optimum set of feed and depth of cut which gives a balance between maximum production rate and minimum production cost as well as provides desired tool life. Such problem for turning operation have been attempted by Ravindran [9] and Sundram [10] using multigoal approach.

In fine surface grinding operation, the optimum set of cutting parameters should be such that not only the total cost of grinding per piece is minimized but also certain desired levels of grinding wheel life (useful life between successive wheel dressings) and metal removal rate are achieved. The desired production rate from the machine will dictate the metal removal rate. On the other hand, putting a lower limit on wheel life will enable the wheel dressing frequency to fit the production time schedule. The metal removal rate and wheel life represents two conflicting goals. By increasing the work velocity or depth of cut, the metal removal rate and hence the production rate can be increased. However, this results in excessive

wheel wear; more frequent dressing and truing of the wheel, forcing more idle time in the production system and hence increased cost of production.

Thus, the determination of optimum grinding conditions involves multiple conflicting goals. In literature, the analysis of grinding efficiency and optimization for precision grinding have been considered for two cases. In the first case wheel life and redressing is an important economic consideration, while in the other case the objective is to maximize removal rate. For the first case, Malkin [11] has used a linear attritious wear model to obtain a relationship for the volume removed per wheel dressing. Taking the total production time as the sum of grinding and dressing times, a relationship was obtained showing the effect of the grinding parameters and dressing time on the production time per unit volume of material removal. A similar approach was taken by Trmal and Kaliszer [12] , the main difference being that the volume removed per wheel dressing entered through empirical relationships for the deterioration in surface finish and increase in grinding forces with volume of material removed. In many situations, especially with short grinding cycles or automated rotary dressing, wheel life may not be an important practical factor, and then objective is to find the maximum removal rate subject to constraints on such factors as

surface finish, metallurgical damage and chatter. Mayne and Malkin [13] obtained results for the maximum allowable removal rate when grinding steels subject to constraints due to workpiece burn and surface finish. They also found the need of adding wheel life between dressings as another performance factor to complete the grinding optimization problem. Maris, Snoeys, and Peters [14] developed grinding charts illustrating the effect of grinding conditions on grinding forces, material removed per wheel dressing, grinding ratio (volume ratio of material removed and wheel wear) and surface finish. By placing the lower limits on these parameters, it should be possible to identify the optimum grinding conditions corresponding to maximum metal removal rate or minimum cost. In an analogous manner, Hahn and Lindsay [15] introduced surface integrity limitations while maintaining high production rates in controlled force grinding. Hahn [16] also developed a computer programme which can be used to estimate the optimum grinding cycles in controlled force internal plunge grinding. De Filippi [17] described an economic analysis of the vertical spindle abrasive machining. As mentioned earlier the determination of optimum grinding conditions involves the balancing of conflicting multiple goals, hence there is a need to go for a more rational approach for the selection of optimum grinding parameters. The only alternate method to the numerical approach for problems involving multiple conflicting objective criteria is the ordinal solution approach.

Goal programming based on the ordinal solution approach appears to be the most appropriate, flexible, and powerful technique for complex decision problems involving multiple conflicting goals.

1.2 THE GRINDING PROCESS

Grinding is one of the most precise and technically important material removal operation. In past it was considered to be a method of finishing hard surfaces only, but today it is being used to remove large stock of metals of all hardnesses and has found an important place in manufacture and its scope is constantly increasing. In the grinding process, material removal is achieved by means of a rotating abrasive wheel, in which a large number of abrasive grains are held together by means of a bonding agent. In comparision to other machining processes, the mechanics of grinding is complex due to random geometry and distribution of cutting points, high cutting speed and variation in geometry of cutting points continuously and unpredictably due to wear during grinding. The grinding operations can be broadly classified as Form and Finish Grinding (FFG) and Stock Removal Grinding (SRG). For the purpose of optimization a typical example from each of these categories have been considered, viz. horizontal surface grinding (FFG) and vertical surface grinding (SRG).

1.2.1 Horizontal Surface Grinding

A typical example of Form and Finish Grinding is the horizontal surface grinding operation (Fig. 1.1 (a)). This operation is used to achieve fine surface finish. Here the material removal rate is limited, chips are smaller in size and wear is usually confined to the grain tips. Under the action of the cutting forces and high temperatures, the wheel loses its cutting ability due to wear and loading. As a result the surface quality of the ground component deteriorates and thermal damage may occur. To prevent these, wheel needs frequent dressing and truing. Major wheel loss is during dressing rather than in grinding. Main input parameters are wheel velocity, work velocity, depth of cut and wheel diameter. Finish grinding wheels have finer grains (Mesh size 46 to 100) and wheel life is the main factor determining their performance. Wheel life may be limited by forces, grinding burns, surface finish, chatter etc. and it was found to vary inversely with increase in work velocity and depth of cut [18] . Typical force pattern (Fig. 1.3) which follows the wheel wear curve (Fig. 1.2), may be divided into three regions.

1. An unstable region where the forces rise to a peak and then fall to a steady value when finite wear flats are developed on sharp grains.

2. A region of stable grinding condition where forces are constant and heat is in equilibrium. Duration of this region depends upon the reaction of the wheel to the particular combination of table speed and depth of cut.

3. A region of sharp increase of forces where dulling of grains reaches a critical value and overheating develops, showing workpiece burn which is due to austenite formation above 650 °C in case of steel, resulting into discolouration of the work surface. Chip thickness has been found to be an important variable affecting the grinding characteristics and significantly influences the grinding energy, wheel wear, surface finish, grinding temperature etc. [19] .

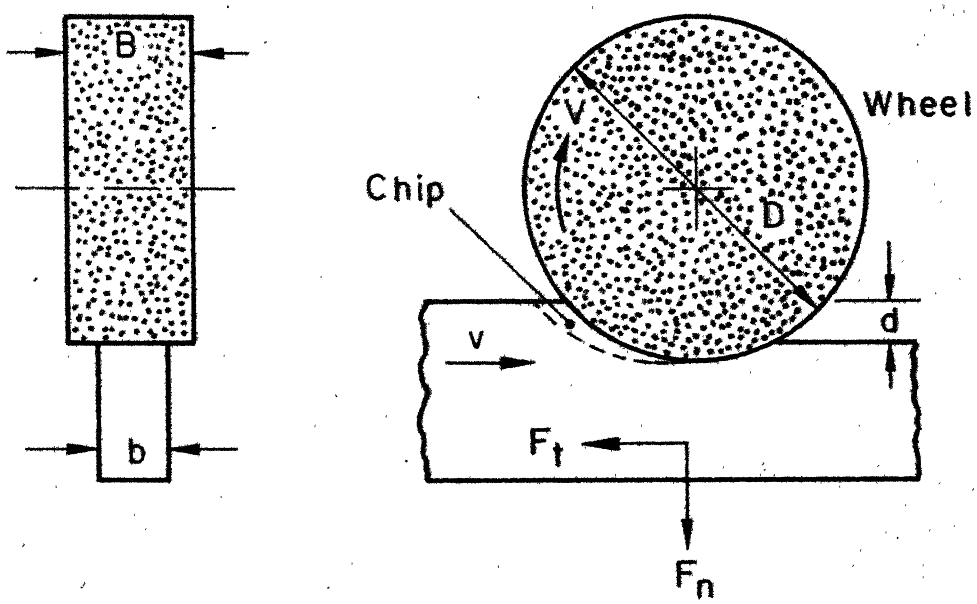
1.2.2 Vertical Spindle Surface Grinding

This is a stock removal process. The metal removal rate is high and the size of individual chips is large. In this process, wheel segments are used instead of a continuous wheel. These segments, after clamping on the hub, are trued usually with the help of a serrated type of wheel crusher made of hard steel. Work table may be either rotory or reciprocating type. Softer Grades (F-I) and coarser grains (Mesh size 24 to 60) are generally used and the wheels are self dressing. The wheel spindle is slightly inclined to

the vertical axis on most of the vertical surface grinding machines which provides a major cutting surface to the workpiece during grinding. The main input parameters are wheel velocity, wheel width, work velocity and depth of cut. Resultant force has components in tangential, normal and longitudinal directions (Fig.1.1(b). Value of longitudinal force is generally much smaller than other two [20] . Grinding ratio is inversely proportional to the metal removal rate and its values are relatively low (1 to 25) [21] . In this grinding operation there is little interest in surface finish. The major concerns are the rate of metal removal and grinding ratio.

1.3 PRESENT WORK

Realizing the necessity of a mathematical model with multiple goals in optimization of parameters in surface grinding and recognizing the capabilities of a goal programming technique, which is specially designed to solve problems involving conflicting multiple goals, this work attempts an application of goal programming for the determination of optimum cutting parameters. Goal programming models for fine surface grinding and vertical surface grinding are developed. The solution methodology is presented and illustrated for the two surface grinding operations considered. A computer code based on the partitioning algorithm [22] has been used for solving the goal programmes.



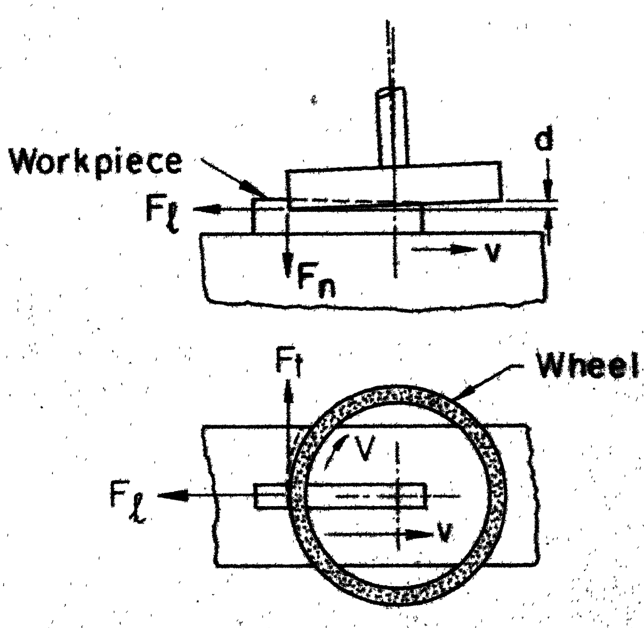


Fig. 1.1 Kinematic arrangements of (a) Horizontal surface grinding (b) Vertical surface grinding.

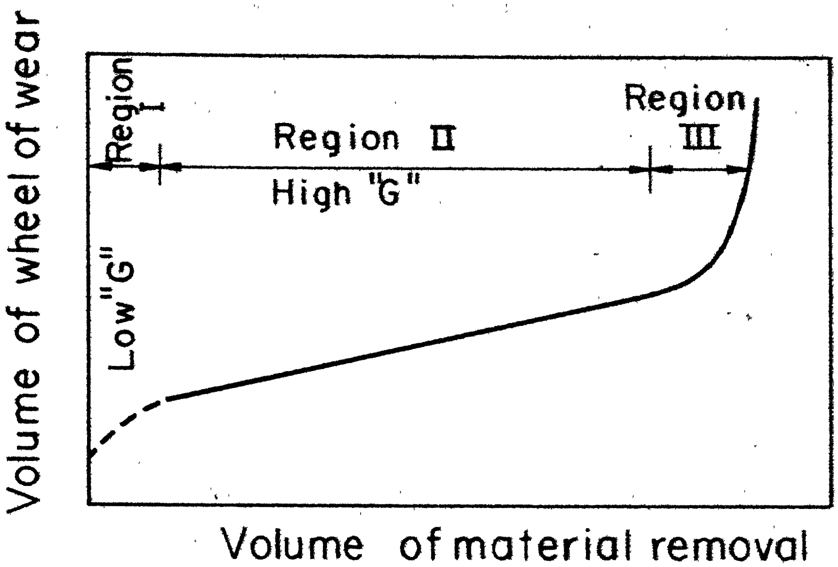


Fig. 1.2 Grinding wheel wear curve

[After Krabacher (37)]

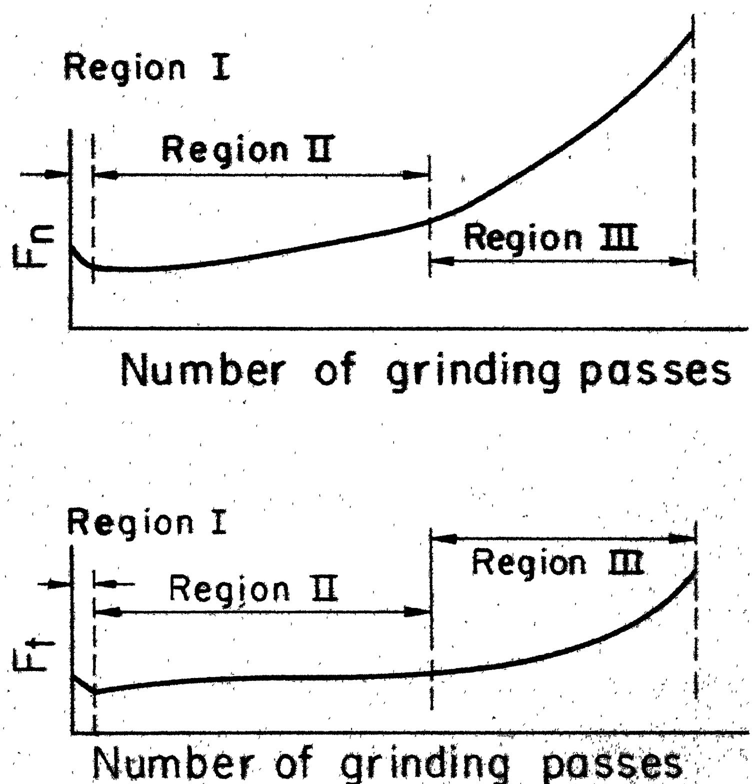


Fig. 1.3 Force pattern during grinding

CHAPTER 2

GOAL PROGRAMMING APPROACH

2.1 INTRODUCTION

Depending upon the nature of objective function and constraints, the problem of economic machining with single objective function, can be solved using any of the suitable Mathematical Programming Techniques e.g., differential calculus approach, Lagrange multipliers, geometric programming, nonlinear programming, linear programming and goal programming. In this chapter, we shall discuss the goal programming method for multiple objective optimization.

2.2 A GOAL PROGRAMMING MODEL

Goal programming is a special type of linear programming which is capable of handling decision situations involving single or multiple goals. A linear programming model has the characteristics of quantifying a single goal in the form of an objective equation. If there are other goals in a linear programming model, they are incorporated in the model as restrictions. In solving such a model, the other goals written in the form of restrictions take precedence over the one in the objective function. That is, the goal in the objective function receives the lowest priority. In the case of goal programming technique all the goals can be incorporated into the objective function

assigning different priorities for each of the goals to be met. This technique has the unique advantage of handling multiple goals in multidimensions over the linear programming model. When goals conflict with each other in a linear programming model, infeasible solution would be the result, whereas the goal programming technique would offer a feasible solution. For example, in a grinding operation the objectives of maximizing the production rate and minimizing the production costs conflict with each other. These two conflicting objectives can be handled assigning different priorities for each one of the goals. Goal Programming technique has been successfully used in aggregate production planning [23], academic resource allocation [24], sales effort allocation and suburban location preference [25].

A goal programming model like an L.P. model does not attempt to maximize or minimize the objective function directly. Rather, the goal programming model seeks to minimize the deviations between the desired goals and the actual results to be obtained according to the assigned priorities. Therefore, the general goal programming model may be written as follows:

Objective function:

$$\text{Minimize,} \quad z = \sum_{i=1}^{m} (d_i^+ + d_i^-)$$

where d_i^+ represents the degree of overachievement of goal i, and d_i represents the degree of underachievement of

goal i. The objective function may also include real variable [26] , which are called the decision variables.

Constraints:

$$\sum_{j=1}^{n} a_{ij}\, x_j - d_i^{+} + d_i^{-} = b_i \qquad \text{for } i = 1, \ldots, m$$

and

$$x_j,\ d_i^{+},\ d_i^{-} \geqslant 0$$

As d_i^{+} and d_i^{-} represent the overachievement and underachievement of the goals, it is not possible to simultaneously achieve both. Hence, one or both of these variables must have a value of zero. In other words, $d_i^{+} \cdot d_i^{-} = 0$ (for all $i = 1,2, \ldots m$). The non-negativity restrictions on the variables as in a L.P. model are also applicable to these deviational variables; that is $d_i^{+}, d_i^{-} \geqq 0$ (for all $i = 1,2,3, \ldots, m$). The solution procedure of a goal programming model is somewhat similar to the simplex method solution for an L.P. model. In goal programming the solution procedure moves the values of these deviational variables as close to zero as possible within the given constraints. Following are some of the important distinguishing features.

The goal programming technique has another unique feature that the conventional linear programming does not have. This is the capability of handling a number of goals with different priorities simultaneously in the objective

function. When many goals with varying levels of priorities are assigned, the model attempts to satisfy the goal with the highest priority first and then tries to meet the goal with the next highest priority. This important feature is useful in situations where conflicting goals are to be included in the model.

When dealing with multiple goals, the deviational variables d_i^+ and d_i^- for each one of the goals are ranked according to their preemptive priority weightsby the model builder, from the most important to least important. Although the method for solving goal programming models involving priorities is similar to the simplex algorithm, there is very slight difference relative to the choice of the entering variable. Variables in the lower priorities are considered for entry only after higher priority variables are no longer available for entry in the solution.

The objective function for such a model with multiple goals and preemptive priorities will be as written below

$$\text{Minimize,} \quad z = \sum_{i=1}^{m} P_i \, (d_i^+ + d_i^-),$$

where d_i^+ is the overachievement of goal i, d_i^- is the underachievement of goal i, and P_i is the preemptive priority factor of goal i. It should be pointed out that the relationship for the preemptive priority factor will be as follows:

$$P_i \gg P_{i+1} \quad (\text{for} \quad i = 1, 2, \ldots, m-1)$$

The objective function can also be written so as to handle different priorities for overachievement and underachievement of the same goal. Weights can be attached to the deviational variables at the same priority level, i.e., variables with the same P_i coefficient.

The general model for the linear goal programming problems can now be stated as:

$$\text{Minimize} \quad z = \sum_{k=1}^{p} P_k \quad (w_{ki}^{-} \, d_i^{-} + w_{ki}^{+} \, d_i^{+}) \tag{2.1}$$

subject to

$$\begin{array}{l}
a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = f_1 \\
a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n = f_2 \\
\vdots \\
a_{l1} x_1 + a_{l2} x_2 + \cdots + a_{ln} x_n = f_l
\end{array} \tag{2.2}$$

$$\begin{array}{l}
e_{11} x_1 + e_{12} x_2 + \cdots + e_{1n} x_n + d_1^{-} - d_1^{+} = b_1 \\
e_{21} x_1 + e_{22} x_2 + \cdots + e_{2n} x_n + d_2^{-} - d_2^{+} = b_2 \\
\vdots \\
e_{m1} x_1 + e_{m2} x_2 + \cdots + e_{mn} x_n + d_m^{-} - d_m^{+} = b_m
\end{array} \tag{2.3}$$

$$x_j \geq 0 \ (j = 1, 2, \cdots, n); \quad d_i^{-}, \ d_i^{+} \geq 0 \ (i = 1, 2, \cdots, m) \tag{2.4}$$

where

x_j : the j^{th} decision variable,

a_{ij} : the coefficient of x_j in the i^{th} real constraint,

f_i : the required level for the i^{th} real constraint,

b_i : the target for goal i,

e_{ij} : the coefficient of x_j in goal i,

d_i^- : the underachievement of goal i,

d_i^+ : the overachievement of goal i,

P_k : the k^{th} ordinal priority factor ($P_k \gg P_{k+1}$),

w_{ki}^- : the weight assigned to d_i^- at priority P_k and

w_{ki}^+ : the weight assigned to d_i^+ at priority P_k

The objective function (2.1) attempts to minimize the weighted sum of the deviational variables (d_i^- and d_i^+) at each **priority.** The set of constraints (2.2) describes the real constraints which must hold at any feasible solution; the set of goal constraints (2.3) relate the decision variables to the target of the goals; and (2.4) give the non-negativity restrictions on all variables. Lee [27] has presented a method and a computer programme for solving linear goal programming problems by modifying the simplex method of linear programs. Recently, Arthur and Ravindran [22] have developed an efficient partitioning algorithm based on the simplex method for solving

linear goal programming problems. Since, the partitioning algorithm has been used in the present work, some of its important features are described in the following section.

2.3 THE PARTITIONING ALGORITHM

Many Goal Programming problems involve real constraints (where no deviations are allowed) along with the usual goal constraints. Since it is necessary to find a basic feasible solution to the real constraints before optimizing the goals, the partitioning algorithm performs a Phase I simplex procedure [27] on the real constraints before considering the goal constraints assigned to priority P_1.

The partitioning algorithm then solves subproblem S_1. The optimal table for this subproblem is examined for alternate optimal solutions. If none exists, it is not possible to optimize the goals of the lower priorities $P_2, P_3, P_4, ---, P_p$. The algorithm then substitutes the values of the decision variables $x_1, x_2, ---, x_n$ in the goal constraints assigned to $P_2, P_3, ---, P_p$ and calculates their levels of achievement. If alternate optimal solutions do exist, the next set of goal constraints (those assigned to priority P_2) and the corresponding terms in the objective function are added to S_1. At this time, the elimination procedure is used to delete all of

the non-basic columns with a positive relative cost value and optimization resumes. The algorithm terminates when a unique optimal solution is found to one of the subproblems or when all priorities have been included and optimized. Fig. 2.1 gives a flow chart of the partitioning algorithm (for further details of the algorithm, see [28] .

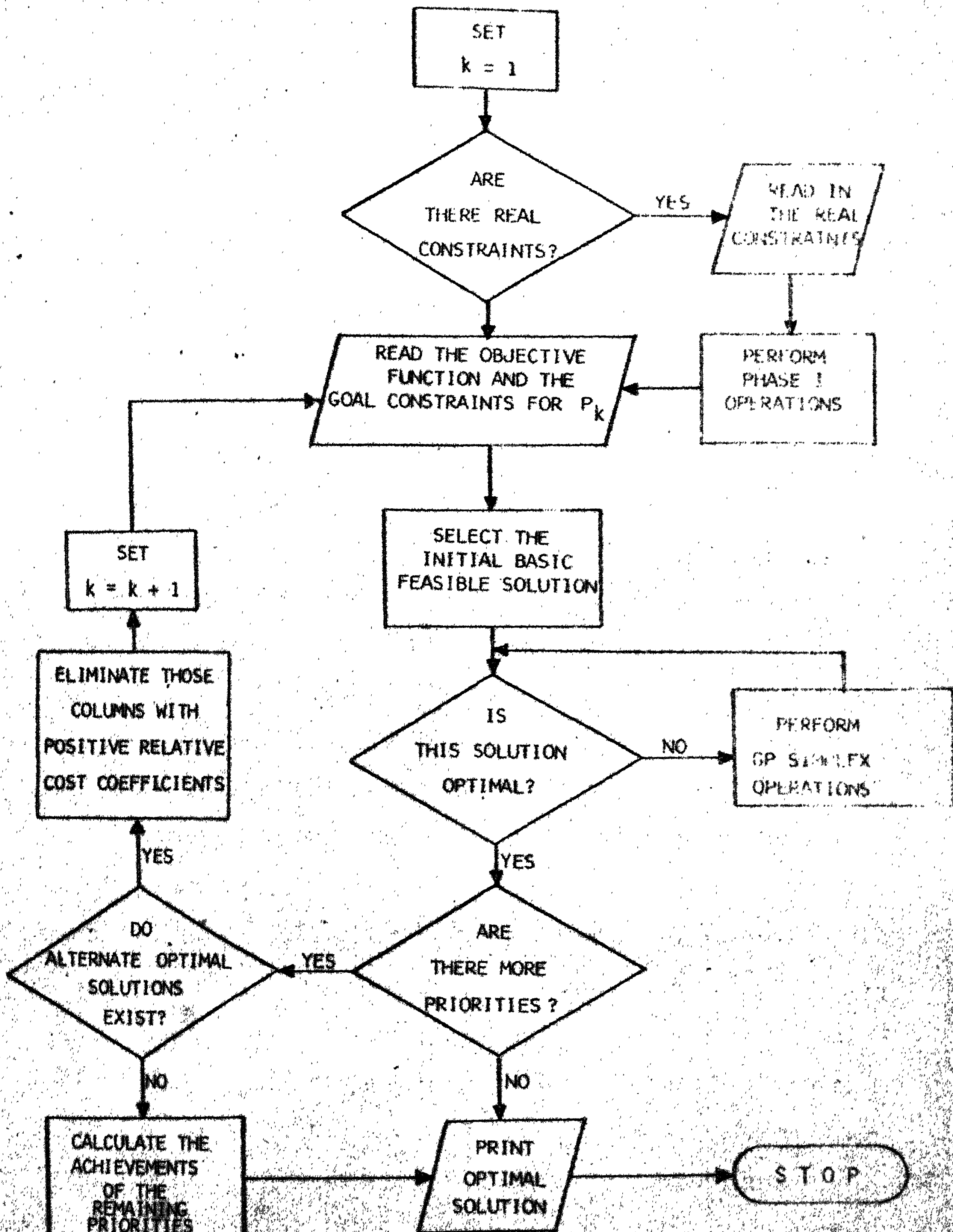


FIGURE 2.1 FLOWCHART OF THE PARTITIONING ALGORITHM

CHAPTER 3

PROBLEM FORMULATION FOR FINE SURFACE GRINDING

3.1 INTRODUCTION

The problem of economics of fine grinding can be formulated as a multiobjective optimization problem. For simplicity plunge surface grinding under dry conditions is considered in this work. Fig. 1.1(a) illustrates its configuration.

The parameters in plunge-cut grinding process are the wheel diameter D, the peripheral wheel velocity V, the work velocity v and the wheel depth of cut d. The performance of the grinding process is governed largely by the choice of these parameters. Most grinding machines are designed such that D and V are fixed and v and d are variable over some range of values. Due to high rates of wheel wear, wheel requires periodic dressing. This reduces the overall output, since dressing is a time-consuming operation and contributes substantially to the total grinding cost. In the goal programming model, we have "real constraints" as well as "goal constraints" (or simply "goals"). The real constraints are absolute restrictions placed on the behaviour of the design variables, while the goal constraints are stated behaviour one would like to achieve but are not mandatory.

The present optimization model considers the multiple objectives in the form of goal constraints as well as real

constraints imposed by machine tool specifications, its dynamic characteristics and the quality requirements of finished component.

3.2 GOAL CONSTRAINT SET

The different grinding process criteria which one may want to consider in a multiple-criteria situation can be categorized as follows:

(a) Goals which refer to specific attainable values of:

- i) metal removal rate
- ii) wheel life
- iii) number of parts produced between successive wheel dressings

(b) Goals to maximize:

- i) metal removal rate
- ii) wheel life
- iii) number of parts produced between successive wheel dressings
- iv) production rate

(c) Goals to minimize cost:

The goals which are considered in this model are minimization of total cost per piece and attainment of certain desired levels of metal removal rate and wheel life.

The equations for the different objectives will now be developed.

3.2.1 Cost Per Piece:

Cost per piece TC in horizontal surface grinding can be expressed as

TC = (cost of non-productive time/piece)
+ (operating cost) + (cost of dressing/piece)
+ (cost of wheel/piece)

Materials costs are not considered. The last three terms may be stated more specifically as

Cost of dressing/piece = (cost of truing + cost of spark out + cost of final dressing)

Cost of wheel/piece = (cost of wheel consumed during truing + cost of wheel consumed during dressing + cost of wheel consumed during grinding)

Operating cost = Cost of labour, overhead and depreciation.

Under plunge grinding conditions used in the present work, the reduction in wheel diameter due to truing and dressing operation was found to be much greater than reduction while grinding, hence this includes the loss of wheel during grinding. The cost equation can be mathematically expressed as

$$TC = C_o T_L + \frac{L_1 \cdot d_1}{d \cdot v} \left[C_o + \frac{C_o B_1}{v_c T} (m + m_1 + m_2) + \frac{C_w \pi B D}{T} (m d_T + d_d) \right] \quad (3.1)$$

where

TC = total cost per piece, paise.

C_o = operating cost per second, paise/sec.

B_1 = traverse length of diamond dresser across wheel face in single pass, cm.

v_c = cross feed rate during truing, dressing operation, cm/sec.

m = number of passes required for truing.

m_1 = number of passes required for sparkout.

m_2 = number of passes required for dressing.

C_w = wheel cost per unit volume, paise.

B = width of wheel, cm.

D = diameter of wheel, cm.

d_T = depth of cut given during truing operation.

d_d = dressing depth of cut.

L_1 = length of cut, cm.

b = width of the workpiece, cm.

d_1 = total depth of material required to be removed, cm.

d = depth of cut given during grinding, cm.

v = work velocity, cm/sec.

T_L = non-productive time (loading, unloading and inspection time) in seconds.

T = wheel-life in secs.

The expression for TC can be written as

$$TC = E + F\, v^{-1}\, d^{-1} + H\, v^{(n_1-1)}\, d^{(n_2-1)} \qquad (3.2)$$

where E, F and H are constants and are given by

$$E = C_o\, T_L \; ; \quad F = C_o\, L_1\, d_1 \quad \text{and}$$

$$H = \frac{L_1\, d_1}{A_1}\left[\frac{C_o\, B_1\, (m + m_1 + m_2)}{v_c} + C_w . \pi\, B\, D\, (m\, d_T + d_d)\right]$$

3.2.2 Wheel Life

Wheel life is an important parameter in evaluating the grinding wheel performance. It reflects not only the physical aspects of grinding but also the economic and technical factors. The different criterion which can be used for establishing the life of grinding wheels are as follows:

a) Limit of dimensional inaccuracy

b) Maximum micro-roughness of the ground surface

c) Maximum power or tangential force

d) Appearance of grinding burn

e) Maximum surface temperature

f) Maximum noise level

g) Maximum amplitude of vibration

In the present work, wheel life data obtained by Pandey [18] from considerations of grinding force, onset of burning and amplitude of vibration has been used to develop empirical relationships between wheel life and grinding

conditions (work velocity and depth of cut). A non-linear regression analysis was performed and the following relationship was obtained for wheel life T in horizontal surface grinding:

$$T = \frac{A}{v^{n_1} d^{n_2}} \tag{3.3}$$

where A, n_1, n_2 are constants, v is the work-velocity in cm/sec., d is the depth of cut in cm., and the wheel life T is in seconds. The details of the analysis are presented in Appendix I - Table I.1.

3.2.3 Metal Removal Rate (MRR)

This is the primary performance measure of the grinding process and can be evaluated from

$$\text{MRR } (\text{cm}^3/\text{sec}) = b \cdot v \cdot d \tag{3.4}$$

where b is the width of workpiece in cms.

3.3 REAL CONSTRAINT SET

The following real constraints are considered in the present formulation:

i) Maximum and minimum permissible work velocity

The maximum and minimum permissible values of work velocity are taken as

$$v_{min} \leq v \leq v_{max} \tag{3.5}$$

ii) Maximum and minimum permissible depth of cut per pass:

The maximum and minimum permissible values of depth of cut are taken as

$$d_{min} \leqslant d \leqslant d_{max} \tag{3.6}$$

iii) Maximum Surface roughness (h_{CLAmax}):

The finish produced in fine grinding is the most important output parameter associated with this finishing process. In order to apply the grinding operation effectively the production engineer should appreciate the influence that each of the grinding variables has upon the surface finish produced.

A few attempts have been made to express the roughness of a finished surface analytically. Sato [29] has expressed the centre-line-average (CLA) roughness of a ground surface in terms of the mean spacing of abrasive grains on the wheel face ($\bar{m}$), the mean spacing of scratches on the ground surface measured in a transverse direction ($\bar{b}'$) and the tip radius of the grain (ρ). He assumed the total roughness to consist of two components, one measured in the direction of grinding ($\bar{h}_1$) and the other ($\bar{h}_2$) measured in the transverse direction. Thus

$$\bar{h} = \bar{h}_1 + \bar{h}_2$$

$$= \left(\frac{v}{V}\right)^2 \frac{\bar{m}^2}{4D} + \frac{\bar{b}'^2}{8\rho}$$

where

$$\bar{m} = \frac{1}{c\,\bar{b}'} \tag{3.7}$$

Here c is the mean number of cutting points on the wheel face per unit area.

Shaw [30] claims that Sato's model incorrectly predicts different values of roughness for different tracing directions. In addition, it yields values of $\bar{h}$ which are too small by more than an order of magnitude.

Orioka [31] has presented an analysis of surface roughness in which the variation of the height of individual grain above the wheel surface is taken into account, and in which the grains are assumed to be randomly distributed on the wheel face. Based on the tracer measurements using a knife-edge stylus the author assumes the grain population density (c) to vary parabolically with the distance from the outermost grain in the wheel face. Ignoring vibration, built-up-edge formation and side flow during chip formation, he obtains the following relationship for surface grinding without cross-feed or spark out.

$$\bar{h} = .328 \left(\frac{v}{V.r.c/\bar{D}}\right)^{2/9} H_o^{2/3} \tag{3.8}$$

where H_o is the maximum grain depth for any grain and

r is the ratio of scratch width to scratch depth.

However, the author presents no simple way of determining H_o for a given wheel and ends up assuming it to lie in the range of 1 to 4 microns.

Yang and Shaw [32] derived an expression for centre-line average (CLA) values of the roughness ($\bar{h}$) for surface grinding, assuming no vibrations, no built-up edge and using mean values throughout. The relation for $\bar{h}$ was given as,

$$h_{CLA} = \left(\frac{v}{2\ V\ r\ c/\bar{D}} \right)^{2/3} \tag{3.9}$$

The equation indicates that surface roughness is independent of wheel depth of cut d, as it should be [30]. The quantity r (ratio of scratch width to scratch depth) may be estimated from a taper section of the ground surface. It was suggested that r lies between 10 to 15 [33]. The value of nominal grain density c to be used corresponds to that obtained under dynamic conditions. The model is based on the assumption that all grains are at the same radial position on the wheel surface and generate chips of equal size. The authors claim that their model yields reasonable values of $\bar{h}$ and this relationship has been considered in the present work. This gives the following additional constraint on the work-velocity

$$v \leqslant (2 \cdot r \cdot c/\bar{D})\ h_{CLAmax}^{3/2} \tag{3.10}$$

iv) Maximum wheel-work interface temperature:

A major limitation in precision grinding of steels is the workpiece burn. At the onset of workpiece burn, the grinding force and rate of wheel wear increase sharply and the surface quality deteriorates. Hence the grinding process should be performed in such a manner that workpiece burn does not occur. The onset of burning is associated with elevated temperature causing discoloration of the finished surface of the workpiece. This puts an upper bound, θ_u, on temperature, i.e.,

$$\theta_{max} \leqslant \theta_u \tag{3.11}$$

Assuming no convective cooling from the workpiece surface and a varying intensity moving heat source acting on the surface, Zerkle [34] has given the following equation from which the wheel-work interface temperature, θ_{max}, can be evaluated

$$\theta_{max} = \frac{2\ K_1 \sqrt{k/\pi}}{K\ .\ b\ (D)^{0.25}} \cdot \frac{F_t\ V\ d^{-.25}}{v^{0.5}} \tag{3.12}$$

Here, b is the width of workpiece, K_1 is the fraction of grinding energy going into workpiece, k is the thermal diffusivity, K is the thermal conductivity and F_t is the tangential force.

An equation for F_t was developed for a given wheel-work combination using extensive experimental data presented by Pande [18]. The non-linear regression

analysis gave the equation in the following form

$$F_t = K_t \, v^{\alpha} \, d^{\beta} \quad (Kg) \tag{3.13}$$

where α, β and K_t are constants, v is in cm/sec. and d is in cm. (The details of the analysis are given in Appendix I Table I.2).

Malkin [35] found that burning threshold is a critical temperature phenomenon. Further, the critical temperature of interest for surface integrity is not the peak temperature generated at the individual cutting points, but some value representative of the overall grinding zone temperature. He has established a relationship for estimating the magnitude of the critical temperature rise and has found it to be 650 °C for plain carbon steel.

v) Maximum horsepower available:

If P_{max} is the maximum horsepower available at the spindle, then

$$v^{\alpha} \cdot d^{\beta} \leqslant \frac{\eta \cdot P_{max} \cdot 4500}{K_t \cdot V} \tag{3.14}$$

where η is the efficiency of machine tool drive. The constants K_t, α and β can be obtained from tangential force equation (3.13).

vi) Non-negativity restrictions on work-velocity and depth of cut

$$v, d \geqslant 0 \tag{3.15}$$

3.4 STATEMENT OF MULTIGOAL OPTIMIZATION PROBLEM

A general multiobjective optimization problem can be stated as:

Find $\bar{x}$ which minimizes $f_1(\bar{x}), f_2(\bar{x}), \ldots, f_n(\bar{x})$ subject to

$$g_k(x) \leq 0, \quad k = 1, 2, \ldots, m$$

where $\bar{x}$ is a set of decision variables (grinding parameters, viz., workvelocity, depth of cut etc.)

$f_i(\bar{x})$, $i = 1, 2, \ldots, n$, are n objective functions

$g_k(\bar{x})$, $k = 1, 2, \ldots, m$, are m constraint functions.

Returning to the machining problem with competing objectives, suppose the management considers that a fine grinding operation will be operating at an acceptable efficiency if the following goals are met in the specified hierarchy as closely as possible:

1. Minimize the total cost per piece
2. The metal removal rate must be greater than or equal to a given rate M_1 (cm^3/sec.)
3. The wheel life must be greater than or equal to T_1 (seconds).

In the goal programming approach the above stated goals are expressed as constraints. Let us consider the total cost of grinding per piece as the first goal. Equation (3.2) gives the following expression for total

cost of grinding per piece,

$$TC = E + F v^{-1} d^{-1} + H v^{(n_1-1)} d^{(n_2-1)}$$

The goal constraint to minimize the cost of grinding can be written as

$$F v^{-1} d^{-1} + H v^{(n_1-1)} d^{(n_2-1)} + d^-_{cost} - d^+_{cost} = 0 \tag{3.16}$$

The above nonlinear goal constraint can be transformed into the equivalent linear goals. It has been found that for most practical problems [3, 36] (a) the optimal solution will lie along the v_{max} constraint, and (b) it will correspond to the points where $\frac{\partial TC}{\partial d} = 0$.

It follows that the optimal v, d combinations can be found by goal programming by first minimizing the deviations from a goal constraint coinciding with v_{max} constraint and then minimizing both deviational variables associated with a goal representing the locus of points along which $\frac{\partial TC}{\partial d} = 0$. Thus, the goals to minimize the cost are

$$v + d_1^- - d_1^+ = v_{max} \quad (v_{max} \text{ goal}) \tag{3.17}$$

and

$$v^{n_1} d^{n_2} + d_2^- - d_2^+ = \text{constant} \quad \left(\frac{\partial TC}{\partial d} = 0 \text{ goal} \right) \tag{3.18}$$

where d_1^- represents the amount by which the goal is underachieved, and d_1^+ represents any overachievement of the goal.

Similarly, metal removal rate and wheel life goals can be expressed as

$$v \cdot d \cdot b + d_3^- - d_3^+ = M_1 \tag{3.19}$$

and

$$\frac{A}{v^{n_1} d^{n_2}} + d_4^- - d_4^+ = T_1, \text{ respectively.} \tag{3.20}$$

Since the first objective is to minimize both deviations from v_{max} goals, the objective must be set-up so that penalties must be assigned to both underachievement as well as overachievement from the goals. Same thing applies to $(\frac{\partial TC}{\partial d} = 0)$ goal. In order to have a metal removal rate of at least M_1, the objective function should include a higher penalty for underachievement of variable d_3^-. No penalty will be assigned to d_3^+. Similarly, to achieve a wheel life of greater than or equal to T_1 penalties must be associated with d_4^- to minimize it to the fullest extent. Accordingly, the goal programming objective function for this problem is

$$\begin{aligned}\text{Minimize } Z &= P_1 (d_1^+ + d_1^-) + P_2 (d_2^+ + d_2^-) \\ &\quad + P_3 d_3^- + P_4 d_4^-\end{aligned} \tag{3.21}$$

where P_1, P_2, P_3 and P_4 are non-numerical preemptive priority factors such as

$$P_i \gg P_{i+1} \quad (\text{for } i = 1, 2, \ldots, 4).$$

While minimizing the objective function, every effort will

be made to completely satisfy the first goal before any attempt is made to satisfy the second goal.

In order to express the problem as a linear goal programming problem, M_1 is replaced by M_2, where

$$M_2 = \frac{M_1}{b} \tag{3.22}$$

and the goal T_1 is replaced by T_2, where

$$T_2 = \frac{A}{T_1} \tag{3.23}$$

and logarithms are taken of the goals and constraints. The various goals and constraints can now be represented as

$$\begin{aligned} \text{Minimize} \quad z = {} & P_1 \, (d_1^+ + d_1^-) + P_2 \, (d_2^+ + d_2^-) + P_3 \, d_3^- \\ & + P_4 \, d_4^- \end{aligned} \tag{3.24}$$

Subject to

(v_{max} goal)

$$\log v + d_1^- - d_1^+ = \log v_{max}$$

($\frac{\partial TC}{\partial d} = 0$ goal)

$$n_1 \log v + n_2 \log d + d_2^- - d_2^+ = \log \text{constant}$$

(MRR goal)

$$\log v + \log d + d_3^- - d_3^+ = \log M_2$$

(wheel life goal)

$$n_1 \log v + n_2 \log d + d_4^- - d_4^+ = \log T_2$$

(3.25)

(v_{max} constraint)

$$\log v \leq \log v_{max}$$

(v_{min} constraint)

$$\log v \geq \log v_{min}$$

(d_{max} constraint)

$$\log d \leq \log d_{max}$$

(d_{min} constraint)

$$\log d \geq \log d_{min}$$

(surface finish constraint)

$$\log v \leq \log \text{constant}$$

(workpiece burn constraint)

$$(\alpha - \tfrac{1}{2}) \log v + (\beta - \tfrac{1}{4}) \log d \leq \log \text{constant}$$

(H.P. constraint)

$$\alpha . \log v + \beta . \log d \leq \log \text{constant} \qquad (3.26)$$

$$\log v_1 \log d,\ d_1^-,\ d_1^+,\ d_2^+,\ d_2^-,\ d_3^+,\ d_3^-,\ d_4^+,\ d_4^- \geq 0$$

(3.27)

CHAPTER 4

PROBLEM FORMULATION FOR VERTICAL SURFACE GRINDING

4.1 INTRODUCTION

The parameters in reciprocating table vertical surface grinding are the peripheral wheel velocity V, the wheel width B, the diameter of wheel D, the work velocity v and the depth of cut d. In the present model, V, B and D are fixed and v and d are variable over some range. The goals to be considered are the minimization of total cost of grinding per piece and achievement of desired levels of metal removal and grinding ratio. In vertical surface grinding, the wheel is self-sharpening and hence the cost of dressing is eliminated. An increase in metal removal rate enhances the rate of wheel wear and results in low grinding ratio. Thus, the goals of maximizing metal removal rate and grinding ratio are conflicting in nature and do not correspond to the same operating conditions. In this chapter, various goals and real constraints for the case of vertical surface grinding have been formulated.

4.2 GOAL CONSTRAINT SET

The following goal constraints have been considered:

4.2.1 Total Cost Per Piece

The total cost per component consists of

a) Machining cost: This includes labour, overhead and depreciation costs.

b) Wheel cost: Cost of wheel lost during grinding.

c) Truing and replacement cost: Cost of labour etc. involved in truing and replacement of wheel segments and cost of wheel lost during truing.

d) Cost of non-productive time : inspection. Cost of loading, unloading and

The cost of grinding per component can, therefore, be expressed as

$$C = C_1 + C_g t_L + \frac{d_t}{d}\left[\left(\frac{L + L_o}{v}\right) C_g\right] + \frac{d_t}{d}\left[\left(\frac{L}{v}\right)\left(\frac{v\ b'\ d}{G}\right) C_w\right] \tag{4.1}$$

where

C_1 = truing and replacement cost, paise

C_g = labour, overhead and depreciation cost, paise/sec

C_w = cost of segmental wheel per unit volume, paise/cm^3

d_t = total depth of cut to be removed per piece, cm

t_L = nonproductive time, seconds

L = length of workpiece, cm

L_o = clearance length on stroke, cm

t_g = machining time per piece, seconds

b' = width of workpiece, cm

v = velocity of table, cm/sec

d = depth of cut, cm

G = grinding ratio

Equation (4.1) can be rewritten as

$$C = C_1 + C_2 + \frac{K_1}{v\ d} + \frac{K_2}{(v\ d)^p} \qquad (4.2)$$

where

$$C_2 = C_g\ t_L; \quad K_1 = d_t\ C_g\ (L + L_o)$$

and $$K_2 = \frac{d_t\ C_w\ L\ b'}{K_r}$$

4.2.2 Metal Removal Rate

It is the primary performance measure for stock removal grinding processes and is given by

$$\text{MRR (cm}^3\text{/sec)} = b' \cdot v \cdot d \qquad (4.3)$$

Figure 4.1 shows the variation of tangential force with metal removal rate and indicates two distinct regions. The experimental data have been taken from reference [37]. The slope of the curve represents the specific power. It is observed that below 25 Kg, the slope is 61.5 H.P/cm^3 per second while above 25 Kg the value of slope is 13.9 H.P./cm^3 per second. The experiments [37] also indicate a sharp increase in the wheel wear rate beyond 25 Kg force. Thus

in this region the wheel acts sharper, lowering the specific power slope. This suggests that vertical surface grinding operation for this wheel-work conbination should be carried out in the range of low specific power for efficient cutting.

4.2.3 Grinding Ratio

It is expressed as the ratio of volume of work removal to volume of wheel consumed and is an important criterion for the selection for stock removal wheels. An empirical relation has been found between grinding ratio and grinding parameters, d and v, using the experimental data obtained by Srihari [37]. The non-linear regression analysis gave the equation in the following form (Appendix II Table II-1):

$$G = K_r \, (v \, d)^p$$

or

$$G = K_g (M_R)^p \qquad (4.4)$$

where K_r, K_g and p are constants and M_R is the metal removal rate.

4.3 REAL CONSTRAINTS

The following real constraints are considered in the present formulation:

i) Bounds on the work velocity

The lower and upper bounds on the work velocity are taken as

$$v_L \leqslant v \leqslant v_u \qquad (4.5)$$

ii) Bounds on the depth of cut per pass

The lower and upper bounds on depth of cut are taken as

$$d_L \leqslant d \leqslant d_u \qquad (4.6)$$

iii) Maximum horsepower available:

If Q_{max} is the maximum horsepower available at the spindle, then

$$v^{\alpha_1} d^{\beta_1} \leqslant \frac{\eta_1 \cdot Q_{max} \cdot 4500}{K_{t_1} \quad V} \qquad (4.7)$$

where η_1 is the mechanical efficiency of the machine-tool drive and the constants K_{t_1}, α_1 and β_1 have been obtained from the tangential force equation developed by performing nonlinear regression analysis on the experimental data of Srihari [37] . The equation obtained is in the following form (Appendix II - Table II.2).

$$F_t = K_{t_1} \; v^{\alpha_1} \cdot d^{\beta_1} \qquad (Kg) \qquad (4.8)$$

where v is in cm/sec and d is in cm.

iv) Nonnegativity constraints:

$$v, \; d \geqslant 0 \qquad (4.9)$$

4.4 STATEMENT OF MULTI-GOAL OPTIMIZATION PROBLEM

Suppose that for a given vertical surface grinding operation, it is desired to achieve the following objectives as closely as possible in order of importance.

1. Minimize the total cost per component.
2. The rate of metal removal should be greater than or equal to a specified value M_2 (cm^3/sec).
3. The grinding ratio must be greater than or equal to G_1.

In the previous section we developed an expression for total cost per component given by equation (4.2). The goal constraint to minimize the total cost of grinding can be written as

$$K_1 \, v^{-1} \, d^{-1} + K_2 \, (v \cdot d)^{-p} + d^-_{cost} - d^+_{cost} = 0 \tag{4.10}$$

The goal represented by this equation is nonlinear in nature and in its present form is not compatible with the linear goal programming procedure. However, this goal can be transformed into a linear goal using the following procedure.

It is obvious that the optimal solution will lie along $\frac{\partial c}{\partial d} = 0$ or $\frac{\partial c}{\partial v} = 0$ which corresponds to the same condition for this case. Thus, it follows that the optimal v, d combination can be found by goal programming by minimizing both deviations associated with a goal representing the locus of points along which $\frac{\partial c}{\partial d}$ or $\frac{\partial c}{\partial v} = 0$. Thus, the goal to minimize the cost is

$$v^{(p+1)} \, d^{(p+1)} + d_1^- - d_1^+ = \text{constant} \tag{4.11}$$

where d_1^- represents the amount by which the goal is underachieved, and d_1^+ represents any overachievement of the goal.

Similarly, the metal removal and grinding ratio goals can be expressed as

$$v \cdot d \cdot b' + d_2^- - d_2^+ = M_3 \qquad (4.12)$$

and

$$\frac{K_r}{\bar{v}^p \cdot \bar{d}^p} + d_3^- - d_3^+ = G_1 \qquad (4.13)$$

Since the first goal is to minimize both deviations from $\frac{\partial c}{\partial d}$ or $\frac{\partial c}{\partial u} = 0$ goal, the objective must include penalties for both underachievement and overachievement from the goals. Similarly, to achieve metal removal and grinding ratio goals, penalties must be associated with underachievement to minimize it to the extent possible.

Accordingly, the goal programming objective function for this problem can be written as

$$\text{Minimize } Z = P_1 \, (d_1^+ + d_1^-) + P_2 \, d_2^- + P_3 \, d_3^- \qquad (4.14)$$

where

$$P_1 >> P_2 >> P_3$$

In order to present the problem as a linear goal programming problem, M_3 is replaced by M_4, where

$$M_4 = \frac{M_3}{b'} \qquad (4.15)$$

and the goal G_1 is replaced by G_2, where

$$G_2 = \frac{K_r}{G_1} \qquad (4.16)$$

Logarithms are taken of the goals and constraints. The multi-goal optimization problem can now be represented as

$$\text{Minimize} \quad Z = P_1\,(d_1^+ + d_1^-) + P_2\,d_2^- + P_3\,d_3^- \qquad (4.17)$$

subject to

(cost goal)

$$(p+1)\log v + (p+1)\log d + d_1^- - d_1^+ = \log \text{constant}$$

(MRR goal)

$$\log v + \log d + d_2^- - d_2^+ = \log M_4$$

(Grinding ratio goal)

$$p\log v + p\log d + d_3^- - d_3^+ = \log G_2 \qquad (4.18)$$

(v_{max} constraint)

$$\log v \leqslant \log v_u$$

(v_{min} constraint)

$$\log v \geqslant \log v_L$$

(d_{max} constraint)

$$\log d \leqslant \log d_u$$

(d_{min} constraint)

$$\log d \geqslant \log d_{min}$$

(H.P. constraint)

$$\alpha_1 \log v + \beta_1 \log d \leqslant \log \text{constant} \qquad (4.19)$$

$$\log v,\ \log d,\ d_1^+, d_1^-,\ d_2^-,\ d_2^+,\ d_3^-,\ d_3^+,\ d_4^-,\ d_4^+ \geqslant 0 \qquad (4.20)$$

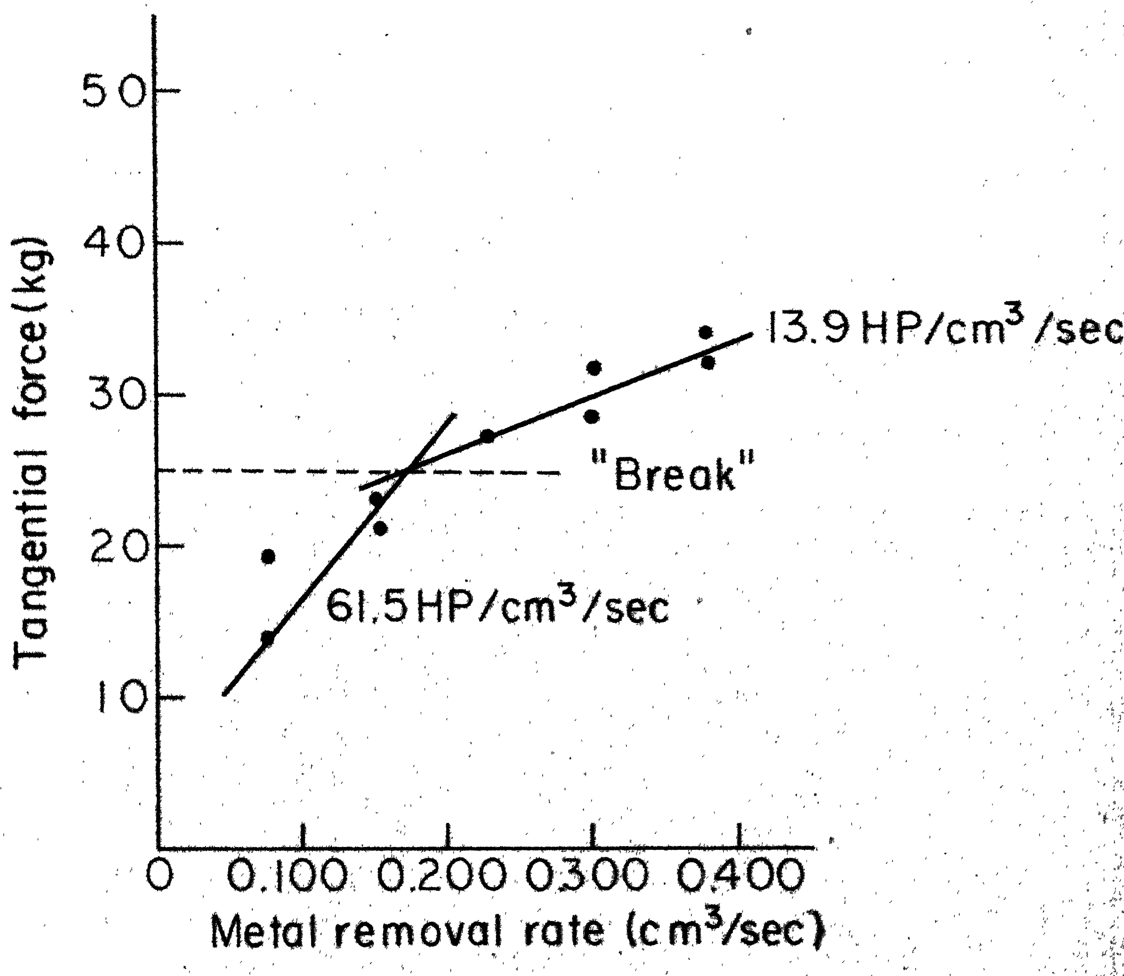


Fig. 4.1 Tangential force vs. metal removal rate in vertical surface grinding (Grain size 24)

CHAPTER 5

SOLUTION METHODOLOGY AND RESULTS

5.1 SOLUTION METHODOLOGY

In the previous chapters, goal programming models for horizontal and vertical surface grinding operations have been developed. For the purpose of illustration a typical example from each of these categories has been presented. A computer code based on the partitioned algorithm developed by Arthur and Ravindran [22] has been used for solving the goal programming problems.

5.2 NUMERICAL EXAMPLE - HORIZONTAL SURFACE GRINDING

The following example illustrates the multiple criteria optimization of a fine surface grinding operation within a production line.

A mild steel workpiece (Hardness 60 R_B) of size 80 x 12 x 25 mm is to be ground under dry conditions to a finish of 0.5 microns (CLA value) on a horizontal surface grinding machine using A 48-I5-V10 grinding wheel. To ensure that this grinding operation is done in such a way that its contribution to the total cost of operating is reduced as much as possible, the following goals, stated in order of importance, have been established:

i) Minimize the cost of operation.

ii) Metal removal rate (MRR) must be greater than or equal to .015 cm^3/sec. to match the system cycle time.

iii) Wheel life must be greater than or equal to 5 minutes for the dressing frequency to fit the schedule established for the system.

The maximum power available on the grinding machine is 3 KW. The wheel diameter and velocity are fixed. With infinitely variable table speeds, the permissible ranges for work velocity and depth of cut on the machine are as follows:

$$4.1667 \leq v \leq 41.667 \quad \text{(cm/sec.)}$$

$$0.0002 \leq d \leq 0.005 \quad \text{(cm/sec.)}$$

The values of other relevant parameters and constants are given in Appendix III.

The other constraints on v and d are as follows:

Using equation (3.10), the surface finish requirements on the workpiece gives the following constraint on v:

$$v \leq 21 \quad \text{(cm/sec.)} \qquad (5.1)$$

For the grinding wheel-workpiece combination under consideration, the tangential grinding force in equation (3.13) is represented as

$$F_t = 23300 \quad v^{0.688} \quad d^{1.38} \quad \text{(Kg)} \qquad (5.2)$$

Substituting the expression for F_t and the relevant constants given in Appendix III in Eq. (3.12) the expression for maximum interface temperature θ_{max} becomes

$$\theta_{max} = 1186801 \ v^{0.188} \ d^{1.13} \tag{5.3}$$

The relationship between the critical grinding energy input at burning and the parameters of the grinding process can be readily incorporated as one of the constraints for the optimization of the surface grinding process.

Malkin [36] has suggested that the interface temperature θ_{max} should be less than or equal to 650 °C to avoid workpiece burns while grinding plain carbon steels. Thus equation (3.11) gives,

$$v^{0.188} \ d^{1.13} \leq 5.4769 \times 10^{-4} \tag{5.4}$$

Assuming an efficiency of 80% for the machine tool drive [18] equation (3.14) gives the power constraint in the following form:

$$v^{0.688} \ d^{1.38} \leq 4.58553 \times 10^{-4} \tag{5.5}$$

Using the values of the parameters given in Appendix III the cost equation (3.1) becomes

$$TC = 10 + v^{-1} \ d^{-1} + 1.439535 \times 10^{6} \ v^{0.1169} \ d^{1.4362} \tag{5.6}$$

To avoid negative values of the variables in log-transformation. the following substitution is incorporated in the model equations:

$$v' = 10000\, v \quad \text{and} \quad d' = 10000\, d$$

Using equations (3.17) and (3.18) the equivalent goals can be represented after logarithmic transformation as

$$\log v' + d_1^- - d_1^+ = 5.61978 \qquad (v_{max} \text{ goal}) \qquad (5.7)$$

$$1.11694 \log v' + 2.4362\, d' + d_2^- - d_2^+ = 7.8971227$$

$$\left(\frac{\partial TC}{\partial d} = 0 \text{ goal} \right) \qquad (5.8)$$

Using equation (3.4), the MRR goal becomes

$$v\, d = 0.0125 \qquad (5.9)$$

On linearizing and introducing deviational variables the MRR goal becomes

$$\log v' + \log d' + d_3^- - d_3^+ = 6.09691 \qquad (5.10)$$

Similarly, the wheel life goal from equation (3.3) reduces to

$$1.11694 \log v' + 2.4362 \log d' + d_4^- - d_4^+ = 7.872938 \qquad (5.11)$$

Thus, the goal programming formulation for this specific case of plunge fine surface grinding can be represented as

$$\text{Minimize} \quad z = P_1 (d_1^+ + d_1^-) + P_2 (d_2^+ + d_2^-) + P_3\, d_3^- + P_4\, d_4^- \qquad (5.12)$$

subject to

$$\log v' + d_1^- - d_1^+ = 5.61978 \quad (v_{max} \text{ goal})$$

$$1.11694 \log v' + 2.4362 \log d' + d_2^- - d_2^+ = 7.8971227$$

$$\left(\frac{\partial TC}{\partial d} = 0 \text{ goal}\right)$$

$$\log v' + \log d' + d_3^- - d_3^+ = 6.09691 \qquad (5.13)$$

$$(\text{MRR goal})$$

$$1.11694 \log v' + 2.4362 \log d' + d_4^- - d_4^+ = 7.872938$$

$$(\text{wheel life goal})$$

$$\log v' \leq 5.61978 \quad (v_{max} \text{ constraint})$$

$$\log v' \geq 4.6198 \quad (v_{min} \text{ constraint})$$

$$\log d' \leq 1.7 \quad (d_{max} \text{ constraint})$$

$$\log d' \geq 0.3011 \quad (d_{min} \text{ constraint}) \qquad (5.14)$$

$$0.688 \log v' + 1.38 \log d' \leq 4.933389$$

$$(\text{H.P. Constraint})$$

$$\log v' \leq 5.3222 \quad (\text{surface finish constraint})$$

$$0.188 \log v' + 1.13 \log d' \leq 2.0105$$

$$(\text{workpiece burn constraint})$$

$$\log v', \log d', d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, d_4^-, d_4^+ \geq 0$$

The above stated problem is solved on DEC 1090 computer system using a FORTRAN-10 code of an improved partitioning algorithm developed by Arthur and Ravindran [22] to solve linear goal programming problems.

The optimal solution is

$$v^* = 21 \text{ cm/sec} \quad (12.6 \text{ m/min})$$

$$d^* = 0.000783 \text{ cm} \quad (7.83 \text{ microns})$$

5.3 NUMERICAL EXAMPLE - VERTICAL SURFACE GRINDING

A vertical surface grinding operation is one of the stages in a flow-type multistage machining system. This operation is to be optimized along with the other stages in the system. Several objectives are identified as being important. These must be met as closely as possible if this operation and the machine group as a whole is to operate at an acceptable level of productivity. It is desired to achieve the following objectives in the specified hierarchy.

i) Minimize the cost of operation

ii) Metal removal rate (MRR) must be greater than or equal to 0.30 cm^3/sec to make the production system cycle time

iii) Grinding ratio must be greather than or equal to 15 to have a satisfactory performance of grinding wheel-work combination.

The component under consideration for grinding is of size 150 x 50 x 25 mm. It is made of mild steel with a hardness of R_B 52. The component to be ground using a grinding wheel with the specification AA - 24 - H - 5 - VL. The maximum power available on the machine is 14 Kw. The wheel

diameter and its velocity are fixed. The values of other relevant parameters and constants are given in Appendix IV. With infinitely variable table speeds, the allowable range for work velocity and depth of cut on the grinding machine are as follows:

$$5 \leqslant v \leqslant 50 \quad (\text{cm/sec.})$$

$$.0005 \leqslant d \leqslant 0.1 \quad (\text{cm})$$

The predictive equations for grinding ratio and force values were obtained from the values shown in Appendix II.

In the goal programming model, restrictions on v and d were modified so that they are not too far away from the experimental range. In view of this the following restrictions on v and d are introduced:

$$5 \leq v \leq 33.33 \quad (\text{cm/sec})$$

$$.001 \leq v \leq 0.1 \quad (\text{cm})$$

The other constraints on v and d are as follows:

For the grinding wheel-workpiece combination under consideration, the tangential grinding force relationship given in equation (4.8) is represented as

$$F_t = 228.3855 \; v^{.312} \, d^{.5194} \quad (\text{Kg}) \tag{5.15}$$

Assuming a mechanical efficiency of 80% for the machine-tool drive [37] the power constraint given by equation (4.7) can be represented as

$$v^{.312} \, d^{.5194} \leqslant .21 \tag{5.16}$$

Substituting the values of the parameters given in Appendix IV, the cost equation (4.1) becomes

$$TC = 20 + .5\ v^{-1}\ d^{-1} + 625\ (v\ .\ d)^{1.5237} \qquad (5.16)$$

To avoid negative values of variables on log-transformation, the following substitution is incorporated in the model equations

$$d' = 10000\ d$$

Using equation (4.11), the equivalent goal can be represented as

$$2.5237 \log v + 2.5237 \log d' + d_1^- - d_1^+ = 6.81494 \qquad (5.17)$$

Using equation (4.3), the metal removal goal becomes

$$v\ .\ d = .06 \qquad (5.18)$$

Linearizing and introducing deviational variables the MRR goal becomes

$$\log v + \log d' + d_2^- - d_2^+ = 2.69897 \qquad (5.19)$$

Similarly, the grinding ratio goal, equation (4.4), gives

$$1.5237 \log v + 1.5237 \log d' + d_3^- - d_3^+ = 3.69686 \qquad (5.20)$$

Thus, the goal programming formulation for this specific cse of vertical surface grinding can be represented as

$$\text{Minimize,} \quad z = P_1 (d_1^+ + d_1^-) + P_2 d_2^- + P_3 d_3^- \qquad (5.21)$$

subject to

$$2.5237 \log v + 2.5237 \log d' + d_1^- - d_1^+ = 6.81498)$$

(cost goal)

$$\log v + \log d' + d_2^- - d_2^+ = 2.69897 \qquad (5.22)$$

(Metal removal goal)

$$1.5237 \log v + 1.5237 \log d' + d_3^- - d_3^+ = 3.69686$$

(Grinding ratio goal)

$$.312 \log v + .5194 \log d' \leq 1.39982$$

(H.P. Constraint)

$$\begin{aligned} &\log v \leq 1.52288 && (v_{max} \text{ constraint}) \\ &\log v \geqslant .69897 && (v_{min} \text{ constraint}) \\ &\log d' \leq 3 && (d_{max} \text{ constraint}) \\ &\log d' \geqslant 1 && (d_{min} \text{ constraint}) \end{aligned} \qquad (5.23)$$

$$\log v,\ \log d',\ d_1^-,\ d_1^+,\ d_2^-,\ d_2^+,\ d_3^-,\ d_3^+,\ d_4^-,\ d_4^+ \geqslant 0 \qquad (5.24)$$

On solving the goal programming model formulated, the following optimum values were obtained for the decision variables:

v^*, the work velocity = 33.33 cm/sec (20 m/min)

d^*, the depth of cut = .0015 cm (15 microns)

5.4 EFFECT OF TABLE SPEED AND DEPTH OF CUT ON COST PRODUCTION

5.4.1 Horizontal Surface Grinding

For the numerical example considered in section 5.2 the effect of table speed and depth of cut on the cost of

production were studied. Fig. 5.1 depicts the variation in cost of production with depth of cut at different table speeds. The effect of table speed at different depths of cut is represented in Fig. 5.2. From the figures it is observed that the minimum cost of production corresponds to table speed of 12.5 m/min and depth of cut of 6 microns.

5.4.2 Vertical Surface Grinding

For the numerical example considered in section 5.3, the effect of table speed and depth of cut on the cost of production were studied. Fig. 5.3 depicts the variation in cost of production with metal removal rate. It is found that there is an optimum metal removal rate (M_R^*) for the minimum cost. The minimum cost of production corresponds to a metal removal rate of 0.3 cm^3/sec.

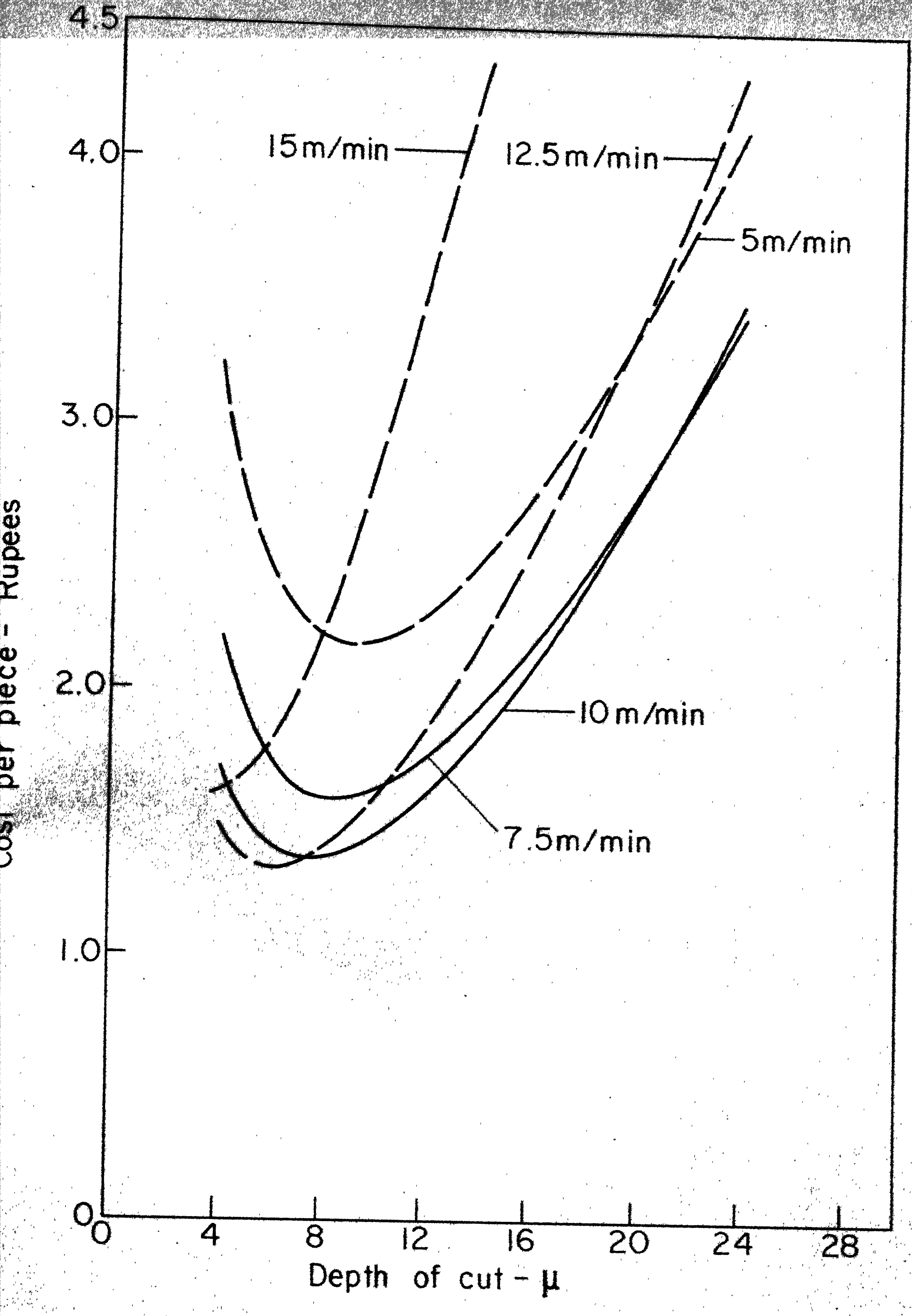
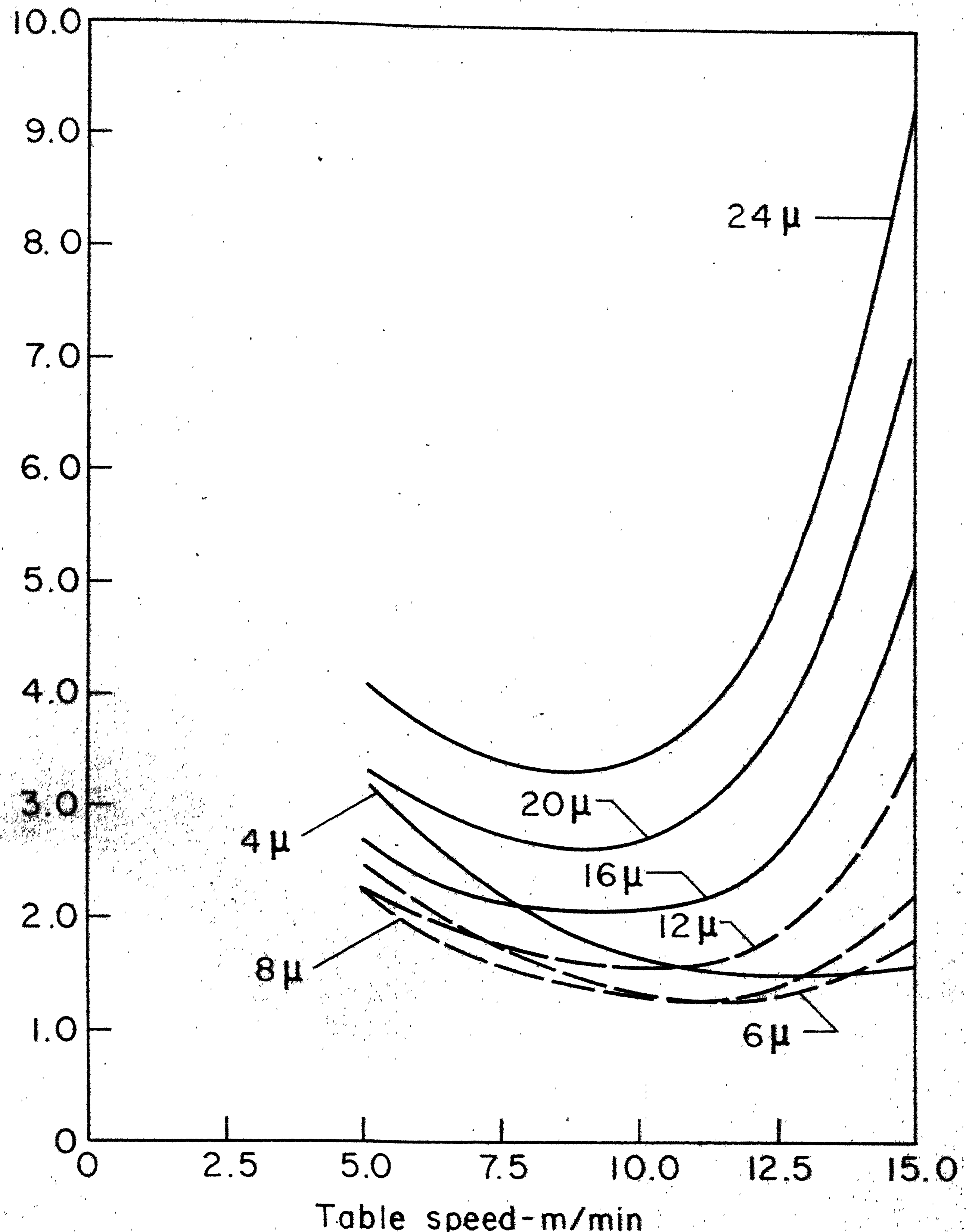


Fig. 5.1 Variation of cost with depth of cut at different table speeds in horizontal surface grinding



ig.5.2 Variation of cost with table speed at different depth of cuts in horizontal surface grinding.

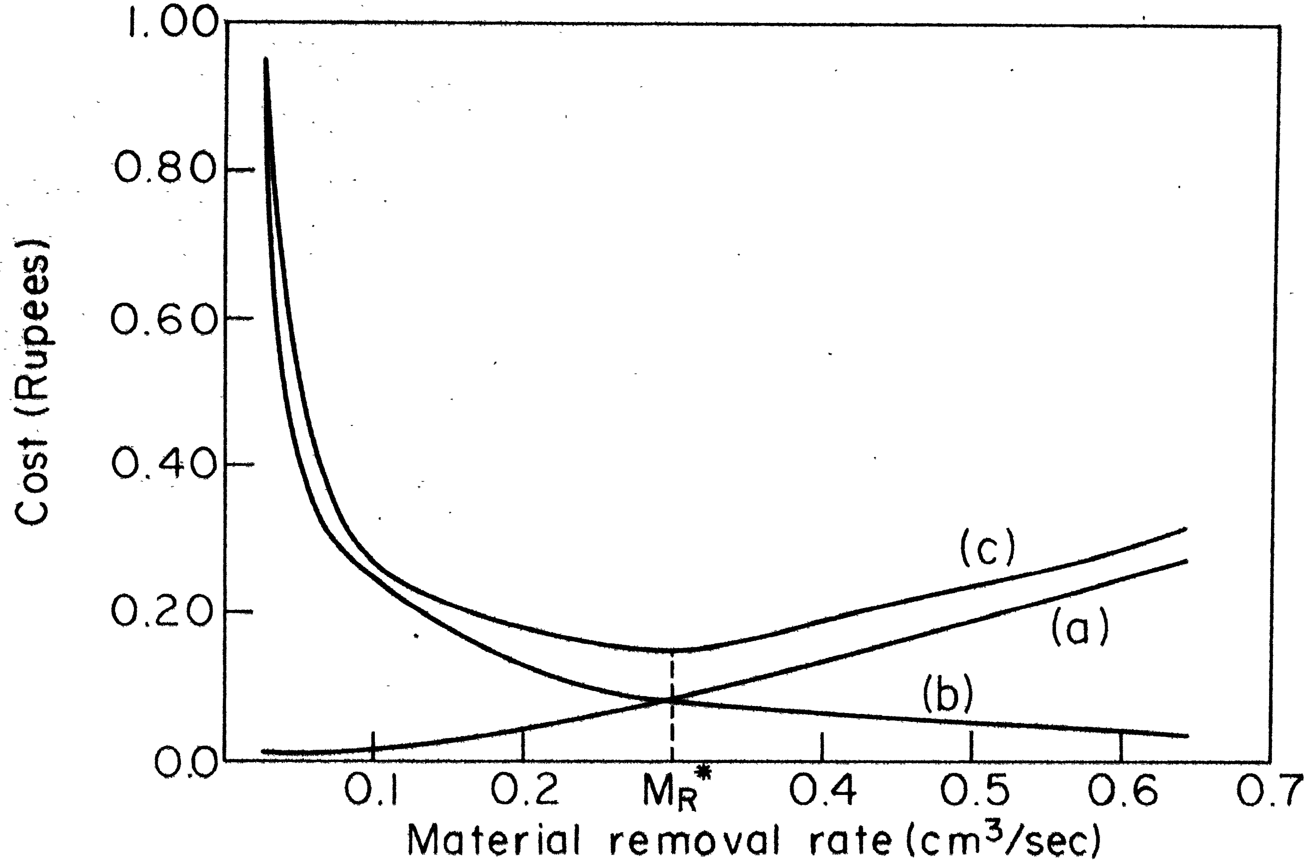


Fig. 5.3 Cost of producing a flat surface on a vertical surface grinding machine (a) machining cost (b) Wheel cost (c) total cost.

CHAPTER 6

CONCLUSION

This thesis demonstrates the use of goal programming technique for the determination of optimal cutting parameters in surface grinding under multiple objective environment. These objectives are generally conflicting in nature and goal programming attempts to achieve the various objectives as closely as possible in a hierarchial fashion. Goal programming models for two specific grinding situations viz., horizontal and vertical surface grinding operations have been developed. The determination of optimum cutting parameters for horizontal surface grinding operation involves consideration of multiple objectives, namely, minimization of machining costs, achieving desired levels of metal removal rate and wheel life. In the vertical surface grinding, the goal of achieving desired level of wheel life is replaced by another performance measure called grinding ratio. All the goals considered in this work can be linearized through logarithmic transformation making the models suitable for the application of linear goal programming. The scope of proposed model can be augmented by introducing more goals and decision variables. For example, width of the work-piece and wheel velocity can be added as additional decision variables. The work can also be extended for the optimization of other manufacturing processes.

REFERENCES

1. Taylor, F.W., "On the Art of Cutting Metals", TRANS. ASME, Vol. 28, 1907, pp. 31-350.

2. Gilbert, W.W., "Economics of Machining", Machining - Theory and Practice, American Society of Metals, 1950.

3. Armarego, E.J.A. and Brown, R.H., The Machining of Metals, Prentice-Hall, Inc., New Jersey, 1969.

4. Bhattacharyya, A., Faria - Gonzalez, R., and Ham, I., "Regression Analysis for Predicting Surface Finish and its Application in the Determination of Optimum Machining Conditions", J. Eng. for Ind., TRANS. ASME, Vol. 92, 1970, pp. 711-714.

5. Brewer, R.C., "Parameter Selection Problem in Machining", Annals of CIRP, Vol. XIV, 1966, pp. 11-17.

6. Ermer, D.S., "Optimization of the Constrained Machining Economics Problem by Geometric Programming", J. Eng. for Ind., TRANS. ASME, Series B, Vol. 93, 1971, pp. 1067-1072.

7. Milner, D.A., "The Use of Linear Programming for Machinability Data Optimization", ASME Paper No. 76-WA/PROD-5, 1976.

8. Hati, S.K. and Rao, S.S., "Determination of Optimum Machining Conditions - Deterministic and Probabilistic Approach", J. Eng. for Ind., TRANS. ASME, Series B, Vol. 98, 1976, pp. 354-359.

16. Hahn, R.S., "Grinding Software - An Important Factor in Reducing Grinding Costs", Proc. Int. Conf. on Prod. Eng., Part 1, Tokyo, Japan, 1974, pp. 650-654.

17. De Filippi, "An Economic Analysis of Vertical Spindle Abrasive Machining", Annals of CIRP, Vol. 23, 1974, pp. 97-98.

18. Pande, S.J., "Wheel Wear Mechanism and Tool Life in Fine Grinding", Ph.D. Thesis, Kanpur University, 1977.

19. Lal, G.K. and Shaw, M.C., "The Role of Grain Tip Radius in Fine Grinding", TRANS. ASME, J. Eng. for Ind., Series B, Vol. 97, 1975, pp. 1119-1125.

20. Lal, G.K., "Forces in Vertical Surface Grinding", Int. J. of Mach. Tool. Des. and Res., Vol. 8, 1968, p. 33.

21. Shaw, M.C., "Grinding Fluids,"Technical Paper No. MR 70-277, Society of Mfg. Engrs. Conf., 1970, Michigan, USA.

22. Arthur, J.L. and Ravindran, A., "A Partitioning Algorithm for (Linear) Goal Programming Problems", Working Paper, School of Industrial Engineering, Purdue University, West Lafeyette, Indiana, Feb. 1977.

23. Lockett, A.G., and Muhlemann, "A Problem of Aggregate Scheduling - An Application of Goal Programming", Int. J. of Prod. Res., 1978, Vol. 16, pp. 127-135.

24. Schroeder, A., "University Resource Planning by Goal Programming", Opns. Res. Vol. 22, 1962, pp. 700-710.

25. Lawrence, K.D. and Burbridge, J.J., "A Multiple Goal Linear Programming Model for Coordinated Production and Logistics Planning", Int. J. Prod. Res., Vol. 14, 1976, pp. 215-222.

26. Lee, Sang M., and Clayton, Edward, R., "A Goal Programming Model for a Academic Resource Allocation", Mgmt. Science, Vol. 18, 1972, pp. B395-399.

27. Lee, Sang M., Linear Optimization for Management, Charter Publishers, New York, Inc., 1976.

28. Arthur, J.I., "Contributions to the Theory and Application of Goal Programming", Ph.D. Dissertation, School of Industrial Engineering, Purdue University, West Lafayette, Indiana, 1977.

29. Sato, K., "Progress of Researches on Grinding Mechanics in Japan", Bull. Japan Soc. of Prec. Engrs., Vol. 5, 1965, pp. 1-25.

30. Nakayama, K., and Shaw, M.C., "Study of the Finish Produced in Surface Grinding - Part 2", Proc. I. Mech.E., London, Vol. 182, 1967, pp. 179-194.

31. Orioka, T., "Probabilistic Treatment of the Grinding Geometry", Bull. Japan Soc. of Grinding Engr., Vol. 1, 1961, pp. 27-29.

32. Yang, C.T., and Shaw, M.C., "The Grinding of Titanium Alloys", TRANS. ASME, Vol. 77, 1977, pp. 645-651.

33. Backer, W.R., Marshall, E.R. and Shaw, M.C., "The Size Effect in Metal Cutting", TRANS. ASME, Vol. 74, 1952, pp. 61-72.

34. Des - Ruisseaux, N.R., and Zerkle, R.D., "Thermal Analysis of the Grinding Process", TRANS. ASME, J.Eng. for Ind., Series B, Vol. 92, 1970, pp. 428-434.

35. Malkin, S., "Burning Limit for Surface and Cylindrical Grinding of Steels", Annals of CIRP, Vol. 27, 1978, pp. 233-236.

36. Philipson, R.H. and Ravindran, A., "Application of Mathematical Programming to Metal Cutting", Mathematical Programming Study, 11, 1979, pp.116-134.

37. Srihari, G., "Mechanics of Vertical Surface Grinding", Ph.D. Thesis, I.I.T. Kanpur, 1980.

APPENDIX I

REGRESSION ANALYSIS ON EXPERIMENTAL DATA FOR HORIZONTAL SURFACE GRINDING

Table I - 1: Wheel Life

Work Velocity (v) m/min	Depth of cut (d) microns	Experimental values of wheel life(T) minutes	Regression values of wheel life minutes	Residual
5.0000	8.0000	7.5000	7.50817	-0.00817
5.0000	12.0000	2.7600	2.79604	-0.03604
5.0000	16.0000	1.3800	1.38730	-0.00730
5.0000	20.0000	0.5400	0.80552	-0.26552
5.0000	24.0000	0.4800	0.51663	-0.03663
7.5000	12.0000	2.0000	1.77770	-0.22230
7.5000	16.0000	1.0800	0.88203	0.19797
7.5000	20.0000	0.4800	0.51215	-0.03215
7.5000	24.0000	0.3600	0.32847	0.03153
10.0000	8.0000	3.4800	3.46180	0.01820
10.0000	12.0000	1.5000	1.28917	0.21083
10.0000	16.0000	0.6600	0.63964	0.02036
10.0000	20.0000	0.4200	0.37140	0.04860
10.0000	24.0000	0.2700	0.23820	0.03180
12.5000	8.0000	2.6300	2.69810	-0.06810
12.5000	12.0000	0.6900	1.00477	-0.31477
12.5000	16.0000	0.48000	0.49853	-0.01853
12.5000	20.0000	0.24000	0.28947	-0.04947
12.5000	24.0000	0.14000	0.18563	-0.04565

STANDARD ERROR OF ESTIMATE - 0.1416

COEFFICIENT OF DETERMINATION - R SQUARE 0.99

COEFFICIENT OF CORRELATION - 0.99713

Comparision of experimental and regression values of wheel life is shown in Figure I-1.

Table I - 2: Tangential Force

Work Velocity (v) m/min	Depth of cut (d) microns	Experimental values of tangential force (F_t)Kg	Regression values of tangential force (F_t)Kg	Residual
5.0000	8.0000	4.2400	5.33889	-1.09889
5.0000	12.0000	9.5600	9.34223	-0.21777
7.5000	8.0000	7.3000	7.05000	0.25000
7.5000	12.0000	12.5700	12.35047	0.21953
10.0000	8.0000	9.1000	8.60000	0.50000
12.5000	8.0000	10.6500	10.03000	0.61728
15.0000	6.0000	7.9200	7.64753	0.27247
15.0000	8.0000	10.7000	11.37450	-0.67450

STANDARD ERROR OF ESTIMATE 0.86

COEFFICIENT OF DETERMINATION - R SQUARE 0.96

COEFFICIENT OF CORRELATION - R 0.97873

Comparision of experimental and regression values of tangential grinding force has been shown in Fig. I - 2.

APPENDIX II

REGRESSION ANALYSIS ON EXPERIMENTAL DATA FOR VERTICAL SURFACE GRINDING

Table II - 1 : Grinding Ratio

Work Velocity (v) cm./sec.	Depth of cut (d) cm.	Experimental values of grinding ratio (G)	Regression values of grinding ratio ($\hat{G}$)	Residual
15.25	.00500	3.0000	3.02856	-0.02856
15.25	.00400	5.0000	4.25495	0.74505
15.25	.00300	6.0000	6.59566	-0.59566
15.25	.00200	13.0000	12.23379	0.76621
15.25	.00100	27.5000	35.17451	-7.67451
25.40	.00300	2.5000	3.02856	-0.52856
20.30	.00300	3.5000	4.25495	-0.75495
10.15	.00300	8.0000	12.23379	-4.23379
5.075	.00300	39.5000	36.27452	4.32549

STANDARD ERROR OF ESTIMATE	5.75553
COEFFICIENT OF DETERMINATION - R SQUARE	0.87
COEFFICIENT OF CORRELATION	0.93

Comparision of Experimental and Regression Values of Grinding Ratio has been shown in Fig. II - 1.

Table II - 2: Tangential Force

Work velocity (v) m/min	Depth of cut (d) mm	Experimental values of tangential force (F_t) Kg	Regression values of tangential force (F_t) Kg	Residual
3.0500	0.0100	14.0000	10.48851	3.51149
3.0500	0.0200	16.0000	15.03375	0.96625
3.0500	0.0300	19.5000	18.55787	0.94213
3.0500	0.0400	21.5000	21.54869	-0.04869
3.0500	0.0500	23.5000	24.19666	-0.69666
6.1000	0.0100	13.0000	13.02044	-0.02044
6.1000	0.0200	17.0000	18.66290	-1.66290
6.1000	0.0300	21.5000	23.03775	-1.53775
6.1000	0.0400	25.0000	26.75055	-1.75055
6.1000	0.0500	28.5000	30.03774	-1.53774
9.1500	0.0100	14.0000	14.77613	-0.77613
9.1500	0.0200	23.0000	21.17942	1.82058
9.1500	0.0300	28.0000	26.14418	1.85582
9.1500	0.0400	32.0000	30.35762	1.64238
9.1500	0.0500	34.5000	34.08806	0.41194
12.2000	0.0100	14.0000	16.16358	-2.16358
12.2000	0.0200	22.0000	23.16814	-1.16814
12.2000	0.0300	28.5000	28.59908	-0.09908
12.2000	0.0400	34.0000	33.20815	0.79185
12.2000	0.0500	37.5000	37.28887	0.21113

STANDARD ERROR OF ESTIMATE	1.57469
COEFFICIENT OF DETERMINATION - R SQUARE	0.96000
COEFFICIENT OF CORRELATION - R	0.98037

Comparision of experimental and regression values of tangential force is shown in Fig. II - 2.

APPENDIX III

DATA FOR NUMERICAL EXAMPLE ON HORIZONTAL SURFACE GRINDING

A	=	1.3725716×10^{-4}
n_1	=	1.11694
n_2	=	2.4362
r	=	15
c	=	160 cm^{-2}
k	=	0.138 cm^2/sec
K_1	=	0.8
K	=	0.124 cal/cm. sec ^oC
T_L	=	1 minute
C_o	=	10 paise per minute
C_w	=	5 paise per unit volume
L_1	=	30 cm
d_1	=	2 cm
B_1	=	9.5 cm
v_c	=	1.25 m/min
m	=	5
m_1	=	10
m_2	=	1
B	=	6.3 cm
b	=	1.2 cm
D	=	30 cm
d_T	=	0.002 cm
d_d	=	0.001 cm
V	=	1355 m/min

APPENDIX IV

DATA FOR NUMERICAL EXAMPLE ON VERTICAL SURFACE GRINDING

K_r = 0.06

b' = 5 cm

B = 1760 m/min

C_g = 10 paise/min

C_w = 5 paise per unit volume

d_t = 0.1 cm

L = 15 cm

L_o = 15 cm

t_L = 60 sec

C_1 = 10 paise

p = -1.5237

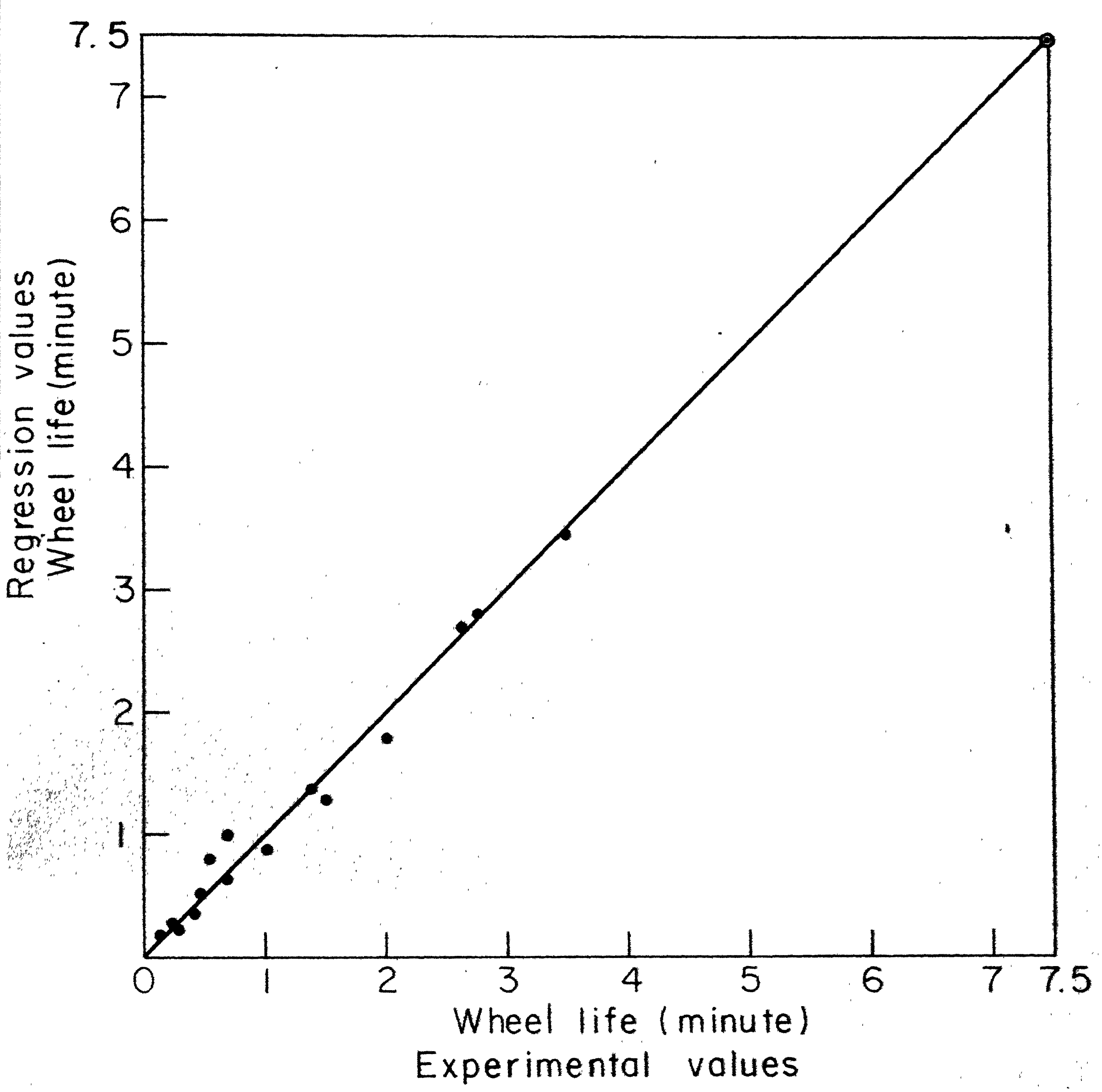


Fig. I-1 Comparison of experimental and regression values - Wheel life in horizontal surface grinding.

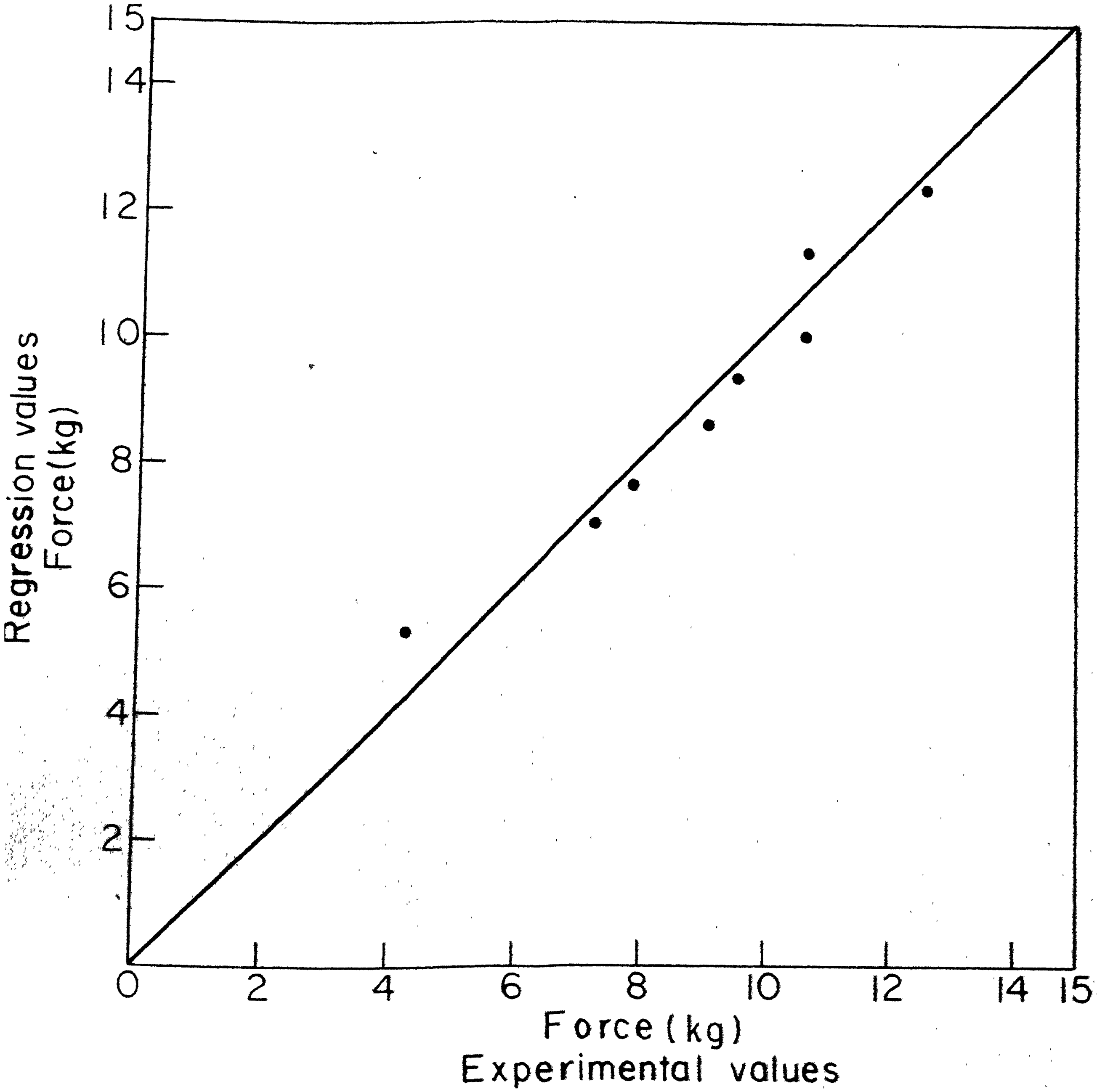


Fig. I-2 Comparison of experimental and regression value- Tangential force in horizontal surface grinding.

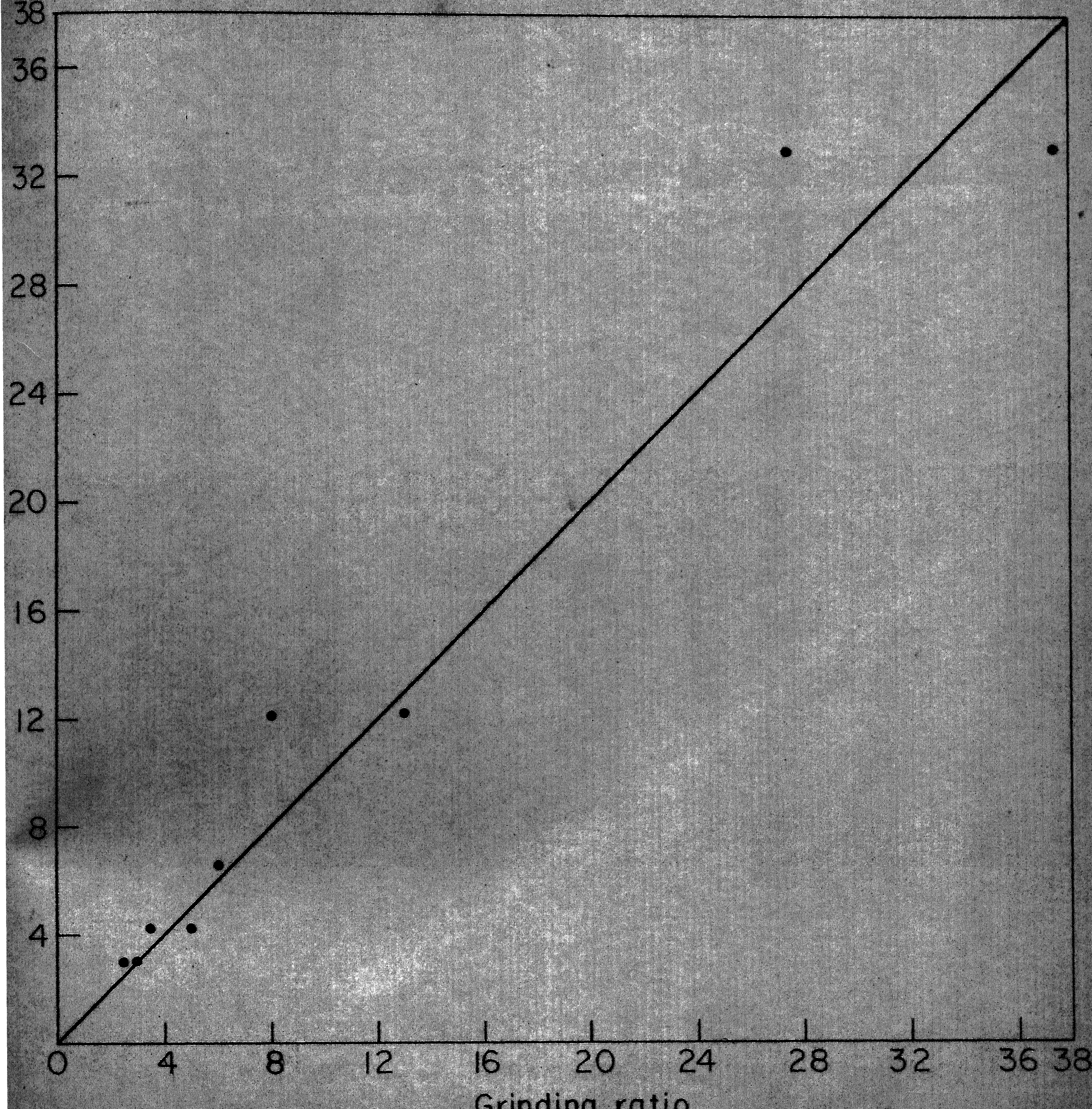


Fig. II-1 Comparison of experimental and regression values - Grinding ratio in vertical surface grinding.

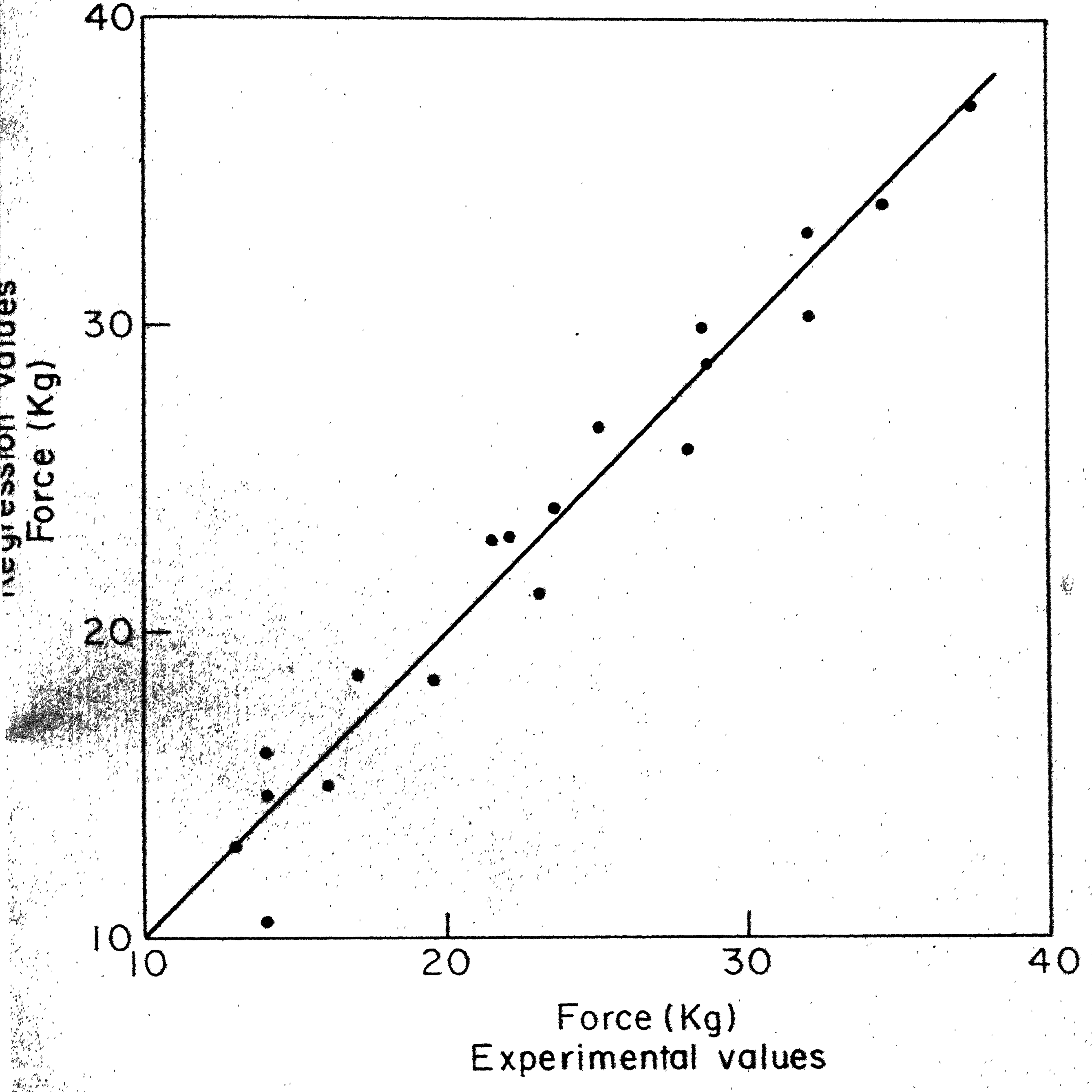


Fig.II-2 Comparison of experimental and regression values- Tangential force in vertical surface grinding.